A P CALCULUS **1st EXAM** **2002** School Holiday 9 / 2	#1 R/C pg 1 - 9 #2 Pg 9 - 10 "Algebra Review" 1 - 15 9 / 3	#3 R/C pg 11 - 15 #4 Pg 15 - 16 1 - 15 18 - 20, 22 - 25 *Be sure your answers are reasonable!*	#5 R/C pg 21 - 25 # 6 Pg 25 - 26 1 - 12, 14 9 / 5	# 7 R/C pg 26 - 27 "Summary" # 8 Pg 27 GQ Ch 2 3 - 8 # 9 Pg 27 - 28 RE Ch 2 1 - 13 "Fenway" 9 / 6
# 10 R/C Pg 29 - 33 R/C Pg 35 - 40 #11 Pg 40 1 - 20 **You *must* use the definition of the derivative!** 9 / 9	**Remind them there are 2 def of der & uses.** # 12 Pg 48 14 - 19 Pg 50 GQ 1 - 7;RE 1-4,7 # 13 R/C "Summary" Pg 48-50 Image = Range pre-image = domain	# 14 Pg 50 - 52 RE 9 - 13, 15 - 17, 19 - 30 **Have pg 50 GQ put on boards and discussed. They need to know this better.** 9/ 11	# 15 Pg 58 - 59 1 - 12, 16 - 18, 20 - 22,25, 29, 30 **"Window on World" Exponents** **Discuss pg 34 18, ~~19~~ also pg 48 pblm 19** 9 / 12	# 16 Pg 64 1 - 5, 9 - 14 **Define *e*** ***Be sure students know that purpose of Window is tht max and min don't always show on the grapher!*** ***lecture in notebook***
Begin "Absolute Bliss" # 17 Parts I and II $\sqrt{\star^2} = \lvert\star\rvert = \begin{cases} -\star & ;\star < 0 \\ \star & ;\star \geq 0 \end{cases}$ 9/16	Continue on "Absolute Bliss" # 18, # 19 Parts III and IV **Have parts I, II put up and explained** 9/17	# 20, 21 "Absolute Bliss" Part V **Have all 10 pblms from lst night put up and explained by students. Hand out more Abs Bliss dittos** 9/18	# 22, #23 "Absolute Bliss" Part VI **involves sgn() and ln() and domain of composition** **Have 19 pblms put and explained.** 9/ 19	# 24, 25 "Absolute Bliss" Part VII **Reasons a limit might not exist:** **"Cannot Approach"** **"Oscillates"** "$\to -\infty$" "$\to +\infty$" **(L) ≠ (R)**
# 26 Pg 71 - 72 1 - 28, 30, 31 # 27 Pg 76 - 77 1 - 30 **limits lecture to include frm above, fm below, etc.** 9/ 23	# 28 Pg 77 31 - 42 **infinite limits** $\lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}} \to$ $\lim_{x\to\infty} \frac{x}{x\sqrt{x+1/x}}$ 9/ 24	REVIEW Put up last year's exam. 9 / 25	**EXAM # 1** **Chapters** **1 --> 4.4** 9/ 26	

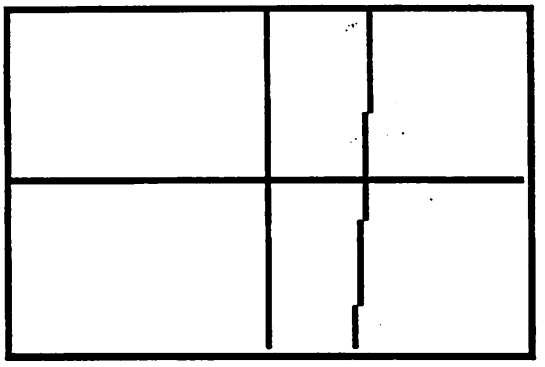
Window on the World

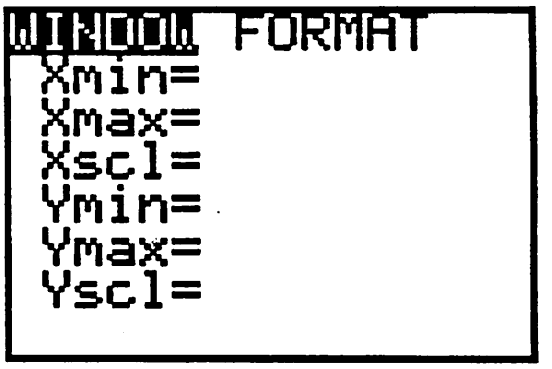
Mr. Mumaugh

Given $f(x) = x^3 - 2x^2 + x - 30$

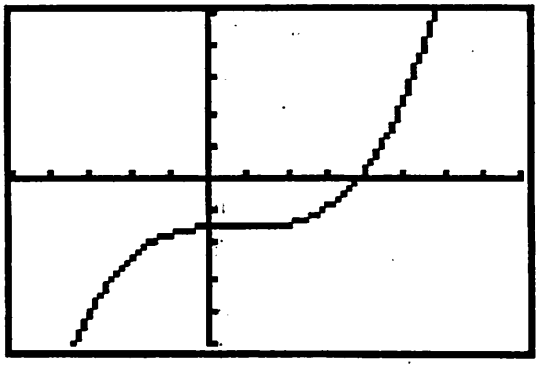
I Determine the "window" settings which gives the supplied picture. Your picture must be exact including the "ticks" on the axes. (hint: YES, these are all of the *same* function as stated above!

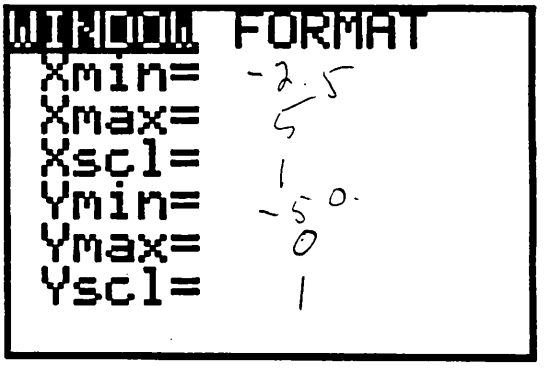
a)



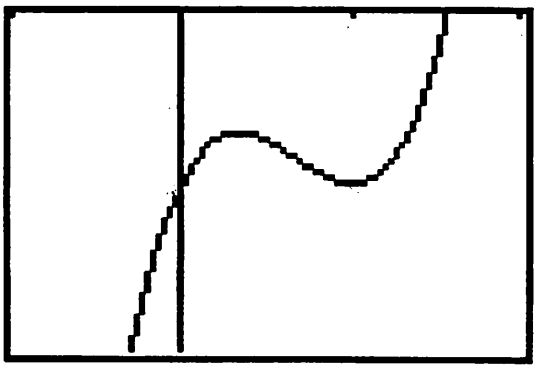


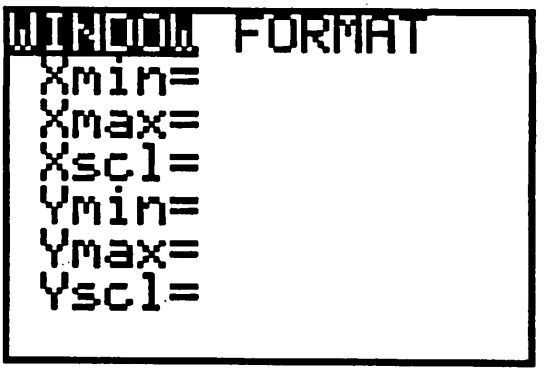
b)





c)





II What are the coordinates of the axis intercepts of the function?

III What are the coordinates of the relative max and the relative min shown in the graph of I c?

IV Why was this particular problem assigned? (What did you learn from it?) Be clear.

Calculus 1st exam

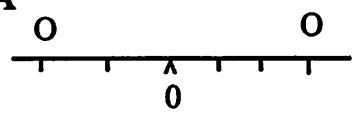
Absolute Bliss

Mr. Mumaugh

Your job is to supply **detailed explanations** for each of the parts A -> E. Copy the problem down the left hand side of your paper. On the right hand side, write your explanations and justifications.

Given: $\left|\frac{x-3}{x+2}\right| < \frac{1}{2}$

A



B $\frac{x-3}{x+2} < \frac{1}{2}$

$2x-6 > x+2$

$x > 8$

Reject because ...

C $\frac{-(x-3)}{x+2} < \frac{1}{2}$

$-2x+6 < x+2$

$4 < 3x$

$\frac{4}{3} < x$

D $\frac{x-3}{x+2} < \frac{1}{2}$

$2x-6 < x+2$

$x < 8$

E conclusion

II

Solve and graph on a number line

A) $|x+3|-|x-2|<3$ B) $|x|+|x|+|x+1|>5$ C) $\left|\frac{x+2}{x-3}\right| < \frac{1}{2}$

III

Re-write each of the following functions as "piece-meal" functions. Graph on the cartesian plane.

A) $F(x)=|2x+3|$ B) $F(x)=|x|+|x+1|$ C) $F(x)=|2x-1|-|2x+1|$ D) $F(x)=\left|x^2+2\right|-\left|x^2-1\right|$

IV

The solution set of: $\frac{x+3}{x-2} \le \frac{x+2}{x-2}$ is

a) $x \in \Re$ b) $x \in \Re,\ x \neq 2$ c) $x<2$ d) $x>2$ e) $x \in \{\ \}$

2) What values of x satisfy the inequality: $\frac{x-1}{x} > \frac{1}{2}$?

a) $x>2$ b) $x<2$ c) $x>\frac{1}{2}$ d) $x<\frac{1}{2}$ e) $x<0$ or $x>2$

3) The solution set of the inequality $|-5x+3| \le 2$ is:

a) $(-\infty, \frac{1}{5}] \cup [1, \infty)$ b) $[\frac{1}{3},\ 1]$ c) $[-1, 1]$ d) $(-\infty, -1] \cup [1, \infty)$ e) $(-\infty, -1] \cup [-\frac{1}{3}, \infty)$

4) The solution set of the inequality $\frac{4x^3-8x^2-3x+9}{(x+1)^3} \ge 0$ is:

a) $[-1,\ \frac{3}{2}]$ b) $(-1,\ \frac{3}{2}]$ c) $(-\infty, -1) \cup [\frac{7}{3},\ \infty)$ d) $(-\infty, -1) \cup (-1,\ \infty)$ e) $[\frac{3}{2},\ \infty)$

5) What values of x make the inequality $|x-2|+x>1$ true?

a) $(2, \infty)$ b) $(-2, \infty)$ c) $(-\infty, \infty)$ d) $(\frac{3}{2}, \infty)$ e) $(-\infty, -\frac{3}{2})$

6) If $|2x-3| < \varepsilon$ then the solution for x in terms of ε is

a) $\frac{3-\varepsilon}{2} < x < \frac{3+\varepsilon}{2}$ b) $x < \frac{3}{2}+\varepsilon$ c) $x < \frac{3}{2}+\frac{\varepsilon}{2}$ d) $x < \frac{\varepsilon}{2}$ e) $\frac{3-\varepsilon}{2} < x < 3+\frac{\varepsilon}{2}$

Absolute Bliss VI

Mr. Mumaugh

Define: $\operatorname{sgn} * = \begin{cases} -1 & \text{if } * < 0 \\ 0 & \text{if } * = 0 \\ 1 & \text{if } * > 0 \end{cases}$ the function $\ln *$ is defined for $*>0$

Determine the domain of the composition of each of the following combinations of equations. You should not compute the actual composition

$$a(x)=(x+2)(x-1) \qquad b(x)=(x-2)(x+1) \qquad c(x)=\frac{x-3}{x^2-6x+9}$$

$$d(x)=\frac{x+3}{x^2+3x+2} \qquad e(x)=\frac{x^2+10x+25}{x+5} \qquad f(x)=\frac{x^2-25}{x+5}$$

A) $A(x)=\dfrac{c(x)}{d(x)}$ B) $B(x)=d(c(x))$ C) $C(x)=c(c(x))$ D) $D(x)=\sqrt{c(x)}$

E) $E(x)=\sqrt{d(x)}$ F) $F(x)=\dfrac{d(x)}{c(x)}$ G) $G(x)=\sqrt{\dfrac{1}{d(x)}}$ H) $H(x)=d(d(x))$

I) $I(x)=\dfrac{\sqrt{c(x)}}{d(x)}$ j) $J(x)=\dfrac{\sqrt{c(x)}}{\sqrt{d(x)}}$ K) $K(x)=\sqrt{\dfrac{c(x)}{d(x)}}$ L) $L(x)=\sqrt{\operatorname{sgn}(a(x))}$

M) $M(x)=\sqrt{\operatorname{sgn}(b(x))}$ N) $N(x)=\ln(a(x))$ O) $O(x)=\ln(d(x))$ P) $P(x)=\ln\left(\dfrac{c(x)}{d(x)}\right)$

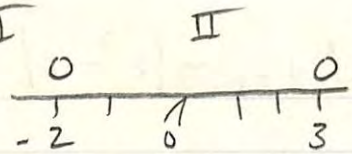
Q) $Q(x)=d(e(x))$ R) $R(x)=e(a(x))$ S) $S(x)=e(e(x))$ T) $T(x)=e(f(x))$

U) $U(x)=f(f(x))$ V) $V(x)=f(e(x))$ W) $W(x)=d(c(c(x)))$ X) $X(x)=\sqrt{a(x)+b(x)}$

Absolute Bliss

I $\left|\frac{x-3}{x+2}\right| < \frac{1}{2}$

A) I II III



B) $\frac{x-3}{x+2} < \frac{1}{2}$ — 1st interval

Notice $x+2 > 0$

$2x-6 > x+2$

$x > 8$

Reject & NOT in first interval

C) 2nd interval

$\frac{-(x-3)}{x+2} < \frac{1}{2}$

$x+2 < 0$ so change inequality

$-2x+6 < x+2$

$4 < 3x$

$\frac{4}{3} < x$

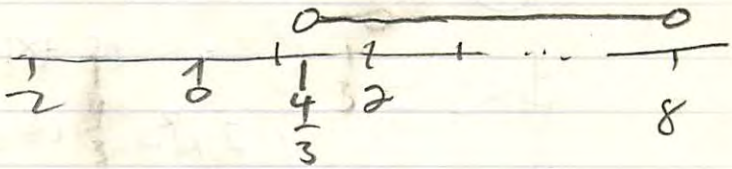
D) 3rd interval

$\frac{x-3}{x+2} < \frac{1}{2}$

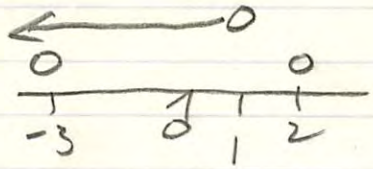
$2x-6 < x+2$

$x < 8$

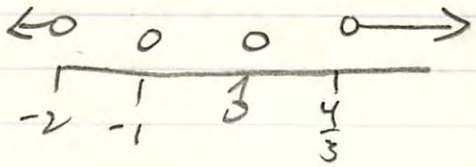
E) conclusion



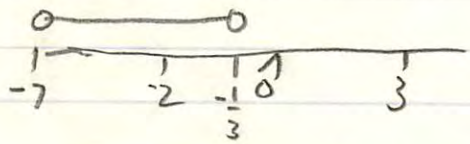
II A) $|x+3| - |x-2| < 3$



B) $|x| + |x| + |x+1| > 5$



C) $\left|\frac{x+2}{x-3}\right| < \frac{1}{2}$



III A) $f(x) = |2x+3|$

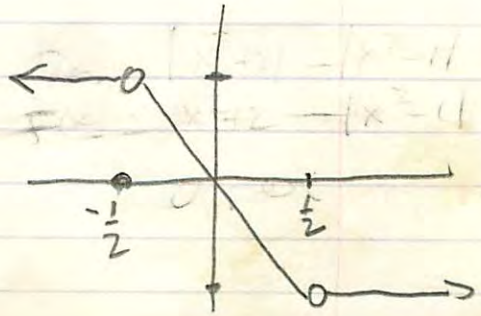
$$f(x) = \begin{cases} -2x+3 & \text{if } x < -\frac{3}{2} \\ 2x+3 & \text{if } x \geq -\frac{3}{2} \end{cases}$$

B) $f(x) = |x| + |x+1|$

$$f(x) = \begin{cases} -2x-1 & \text{if } x < -1 \\ 1 & \text{if } -1 \leq x < 0 \\ 2x+1 & \text{if } x > 0 \end{cases}$$

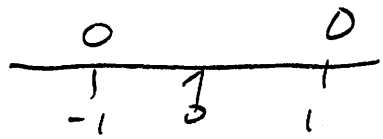
C) $f(x) = |2x-1| - |2x+1|$

$$f(x) = \begin{cases} 2 & \text{if } x < -\frac{1}{2} \\ 0 & \text{if } x \in \left\{-\frac{1}{2}, \frac{1}{2}\right\} \\ -4x & \text{if } -\frac{1}{2} < x < \frac{1}{2} \\ -2 & \text{if } x > \frac{1}{2} \end{cases}$$



III 1) $f(x) = |x^2+2| - |x^2-1|$

① $f(x) = x^2+2 - |x^2-1|$



$x^2+2-(x^2-1)$

x^2+2-x^2+1

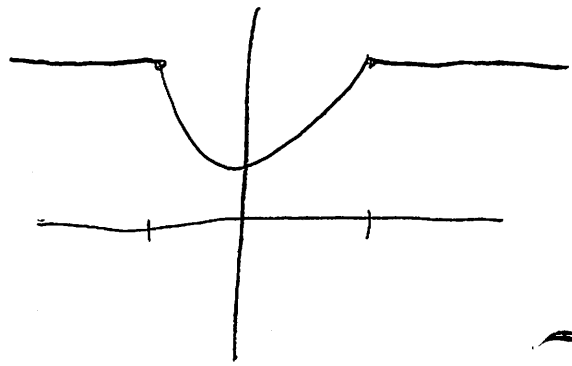
3

② x^2+2+x^2-1

$2x^2+1$

③ 3

$$f(x) = \begin{cases} 3 & \text{if } |x| \geq 1 \\ 2x^2+1 & \text{if } -1 < x < 1 \end{cases}$$



V

Absolute Bliss (continued)

) The equation of a line perpendicular to $x - 3 = 5$ and passing through the point (2, 1) is

a) $x = 2$ b) $y = 1$ c) $x = 8$ d) $y = 2$ e) $x = 1$

) In the linear equation $ax + by = c$ which of the following is necessarily true?

I. The slope of the line is $-\frac{b}{a}$

II. The y-intercept is $\frac{c}{a}$

III. The line contains the point $\left(d, \frac{c - ad}{b}\right)$

a) I only b) II only c) III only d) I and II only e) II and III only

3) The slope of a line perpendicular to the line through the points (3, – 5) and (– 2, – 4) is

a) $-\frac{1}{5}$ b) $\frac{1}{9}$ c) -9 d) $-\frac{1}{9}$ e) 5

4) Find the value of k such that the lines with equations $4kx - 2y = 1$ and $3y + 2kx = -5$ are parallel.

a) $-\frac{2}{3}$ b) 2 c) { } d) zero

5) The length of the longest side of a triangle with verticies (2, 3), (2,1) and (5, 4) is:

a) 2 b) $\sqrt{2}$ c) $3\sqrt{2}$ d) $\sqrt{10}$ e) $\sqrt{19}$

6) Find the value of b so that (3, b), (4, 1), and (--2, 5) are collinear.

a) $\frac{1}{3}$ b) $\frac{2}{3}$ c) 4 d) $\frac{5}{3}$ e) $-\frac{3}{4}$

7) Two lines $ax + by = 3$ and $bx + ay = 2$ intersect at (4, -1) if $a =$

a) $\frac{11}{15}$ b) $\frac{14}{15}$ c) $\frac{17}{30}$ d) $\frac{23}{30}$ e) $\frac{5}{6}$

8) If $f(x) = x^2 + 3x$ and $g(x) = x^3 - x$ then $(f \circ g)(2) =$

a) 990 b) 60 c) 45 d) 54 e) 8

9) Let $f(x) = |3x - 2| + 2|x| - 5$. For values of x in the interval $\left(0, \frac{2}{3}\right)$, $f(x) =$ ...

a) $-x - 3$ b) $5x - 7$ c) $-5x - 7$ d) $x - 7$ e) $x + 3$

10) The domain of the function $f(x) = \frac{3x}{\sqrt{4 - x^2}}$ is:

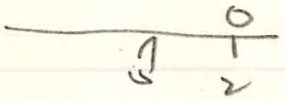
a) $[-2, 2]$ b) $(-2, 2)$ c) $(-2, 2)\,;\, x \neq 0$ d) $[-2, 2]\,;\, x \neq 0$ e) $|x| > 2$

11) Which of the following functions is equivalent to $f(x) = \frac{|x - 2|}{x - 2}$?

a) $f(x) = 1$ b) $f(x) = \begin{cases} -1 & \text{if } x < 2 \\ 1 & \text{if } x \geq 2 \end{cases}$ c) $f(x) = \begin{cases} -1 & \text{if } x \leq 2 \\ 1 & \text{if } x > 2 \end{cases}$

d) $f(x) = \begin{cases} -1 & \text{if } x < 2 \\ 1 & \text{if } x > 2 \end{cases}$ e) $f(x) = 1,\ x \neq 2$

① $\frac{x+3}{x-2} \leq \frac{x+2}{x-2}$

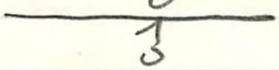


C I $x+3 \geq x+2$
$3 \geq 2$
OK

II $x+3 \leq x+2$
$3 \leq 2$
Reject.

$\therefore x < 2$

② $\frac{x-1}{x} > \frac{1}{2}$

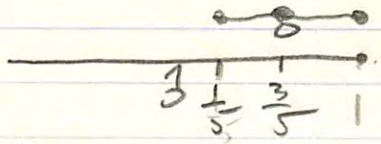


E I $2x-2 < x$
$x < 2$
$\therefore x < 0$

II $2x-2 > x$
$x > 2$

thus $x < 0$ or $x > 2$

③ $|-5x+3| \leq 2$



I $(-5x+3) \leq 2$
B $-5x+3 \leq 2$
$-5x \leq -1$
$x \geq \frac{1}{5}$

II $-(-5x+3) \leq 2$
$5x-3 \leq 2$
$5x \leq 5$
$x \leq 1$

$\therefore \frac{1}{5} \leq x \leq 1$

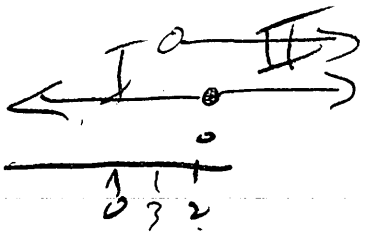
④ $\frac{4x^3-8x^2-3x+9}{(x+1)^3} \geq 0$

$\frac{(x+1)(2x-3)^2}{(x+1)^3} \geq 0$

D

$\frac{(2x-3)^2}{(x+1)^2} \geq 0$ True $\forall x \neq -1$.

4 −8 −3 9
−1 | 4 −12 9 0
$4x^2-12x+9$
$(2x-3)^2$



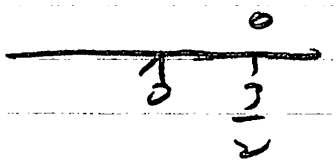
(5) $|x-2|+x>1$

I $-(x-2)+x>1$ II $x-2+x>1$

$2>1$ $2x>3$

(C) always true $x>\frac{3}{2}$ $x\in\mathbb{R}$

(6) $|2x-3|<\varepsilon$



I $-(2x-3)<\varepsilon$

(A) $-2x+3<\varepsilon$

$-2x<\varepsilon-3$

$x>\frac{\varepsilon-3}{2}$

II $2x-3<\varepsilon$

$2x<\varepsilon+3$

$x<\frac{\varepsilon+3}{2}$

Absolute Bliss V

(1) $x-3=5$

$x=8$ Through $(2,1)$ $y=1$

(B) $\perp$ $m=0$

(2) $y=-\frac{a}{b}x+\frac{c}{5}$ I NO II NO

C $-\frac{ad}{5}+\frac{c}{b}$ III Yes.

(3) $(3,-5)$ $(-2,-4)$ $\frac{-4+5}{-2-3}=-\frac{1}{5}$ $\perp$ $m=5$ e

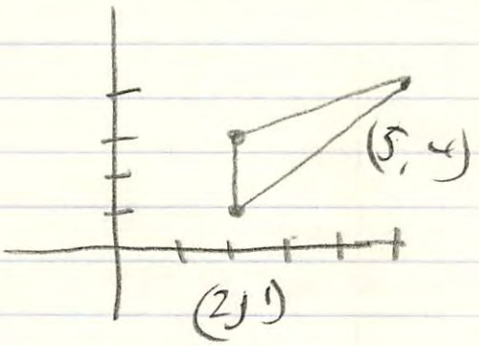
④ $4kx - 2y = 1$ $\quad$ $3y + 2kx = -5$

$y = 2kx - \frac{1}{2}$ $\quad$ $y = -\frac{2k}{3}x - \frac{5}{3}$

D $\quad 2k \overset{\text{swt}}{=} -\frac{2}{3}k$ $\quad$ right C

$\frac{8}{3}k = 0 \quad k = 0$

⑤ $(2,3)\ (2,1)\ (5,4)$



C $\quad \sqrt{(5-2)^2 + (4-1)^2}$

$\sqrt{9+9}$

$\sqrt{18} \quad 3\sqrt{2}$

⑥ $(2,5)$ $(4,1)$ $\quad \frac{5-1}{-2-4} = \frac{4}{-6} = -\frac{2}{3} = m$

$y - 1 = -\frac{2}{3}(x-4)$

$y - 1 = -\frac{2}{3}(3-4)$

$y - 1 = \frac{2}{3}$ $\quad$ ⑩

$y = \frac{5}{3}$

Calculus 1st exam

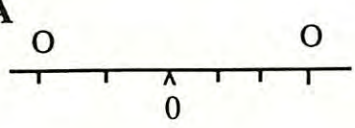
Absolute Bliss

Mr. Mumaugh

I Your job is to supply **detailed explanations** for each of the parts A -> E. Copy the problem down the left hand side of your paper. On the right hand side, write your explanations and justifications.

Given: $\left|\frac{x-3}{x+2}\right| < \frac{1}{2}$

A



B

$$\frac{x-3}{x+2} < \frac{1}{2}$$
$$2x-6 > x+2$$
$$x > 8$$

Reject because ...

C

$$\frac{-(x-3)}{x+2} < \frac{1}{2}$$
$$-2x+6 < x+2$$
$$4 < 3x$$
$$\frac{4}{3} < x$$

D

$$\frac{x-3}{x+2} < \frac{1}{2}$$
$$2x-6 < x+2$$
$$x < 8$$

E conclusion

II Solve and graph on a number line

A) $|x+3|-|x-2|<3$ B) $|x|+|x|+|x+1|>5$ C) $\left|\frac{x+2}{x-3}\right| < \frac{1}{2}$

III Re-write each of the following functions as "piece-meal" functions. Graph on the cartesian plane.

A) $F(x)=|2x+3|$ B) $F(x)=|x|+|x+1|$ C) $F(x)=|2x-1|-|2x+1|$ D) $F(x)=\left|x^2+2\right|-\left|x^2-1\right|$

IV

1) The solution set of: $\frac{x+3}{x-2} \le \frac{x+2}{x-2}$ is

a) $x \in \Re$ b) $x \in \Re,\ x \neq 2$ c) $x<2$ d) $x>2$ e) $x \in \{\}$

2) What values of x satisfy the inequality: $\frac{x-1}{x} > \frac{1}{2}$?

a) $x>2$ b) $x<2$ c) $x>\frac{1}{2}$ d) $x<\frac{1}{2}$ e) $x<0$ or $x>2$

3) The solution set of the inequality $|-5x+3| \le 2$ is:

a) $(-\infty, \frac{1}{5}] \cup [1,\infty)$ b) $[\frac{1}{5},\ 1]$ c) $[-1, 1]$ d) $(-\infty, -1] \cup [1,\infty)$ e) $(-\infty, -1] \cup [-\frac{1}{5},\infty)$

4) The solution set of the inequality $\frac{4x^3-8x^2-3x+9}{(x+1)^3} \ge 0$ is:

a) $[-1,\ \frac{3}{2}]$ b) $(-1,\ \frac{3}{2}]$ c) $(-\infty, -1) \cup [\frac{7}{3},\ \infty)$ d) $(-\infty, -1) \cup (-1,\ \infty)$ e) $[\frac{3}{2},\ \infty)$

5) What values of x make the inequality $|x-2|+x>1$ true?

a) $(2,\infty)$ b) $(-2,\infty)$ c) $(-\infty,\infty)$ d) $(\frac{3}{2},\infty)$ e) $(-\infty,-\frac{3}{2})$

If $|2x-3|<\varepsilon$ then the solution for x in terms of ε is

a) $\frac{3-\varepsilon}{2} < x < \frac{3+\varepsilon}{2}$ b) $x < \frac{3}{2}+\varepsilon$ c) $x < \frac{3}{2}+\frac{\varepsilon}{2}$ d) $x < \frac{\varepsilon}{2}$ e) $\frac{3-\varepsilon}{2} < x < 3+\frac{\varepsilon}{2}$

V

Absolute Bliss (continued)

1) The equation of a line perpendicular to $x-3=5$ and passing through the point (2, 1) is

a) $x=2$ b) $y=1$ c) $x=8$ d) $y=2$ e) $x=1$

2) In the linear equation $ax+by=c$ which of the following is necessarily true?

I. The slope of the line is $-\frac{b}{a}$

II. The y-intercept is $\frac{c}{a}$

III. The line contains the point $\left(d, \frac{c-ad}{b}\right)$

a) I only b) II only c) III only d) I and II only e) II and III only

3) The slope of a line perpendicular to the line through the points (3, – 5) and (– 2, – 4) is

a) $-\frac{1}{5}$ b) $\frac{1}{9}$ c) -9 d) $-\frac{1}{9}$ e) 5

4) Find the value of k such that the lines with equations $4kx-2y=1$ and $3y+2kx=-5$ are parallel.

a) $-\frac{2}{3}$ b) 2 c) { } d) zero

5) The length of the longest side of a triangle with verticies (2, 3), (2,1) and (5, 4) is:

a) 2 b) $\sqrt{2}$ c) $3\sqrt{2}$ d) $\sqrt{10}$ e) $\sqrt{19}$

6) Find the value of b so that (3, b), (4, 1), and (–2, 5) are collinear.

a) $\frac{1}{3}$ b) $\frac{2}{3}$ c) 4 d) $\frac{5}{3}$ e) $-\frac{3}{4}$

7) Two lines $ax+by=3$ and $bx+ay=2$ intersect at $(4,-1)$ if $a=$

a) $\frac{11}{15}$ b) $\frac{14}{15}$ c) $\frac{17}{30}$ d) $\frac{23}{30}$ e) $\frac{5}{6}$

8) If $f(x)=x^2+3x$ and $g(x)=x^3-x$ then $(f\circ g)(2)=$

a) 990 b) 60 c) 45 d) 54 e) 8

9) Let $f(x)=|3x-2|+2|x|-5$. For values of x in the interval $\left(0, \frac{2}{3}\right)$, $f(x)$ = ...

a) $-x-3$ b) $5x-7$ c) $-5x-7$ d) $x-7$ e) $x+3$

10) The domain of the function $f(x)=\frac{3x}{\sqrt{4-x^2}}$ is:

a) $[-2, 2]$ b) $(-2, 2)$ c) $(-2, 2)\,;\, x\neq 0$ d) $[-2, 2]\,;\, x\neq 0$ e) $|x|>2$

11) Which of the following functions is equivalent to $f(x)=\frac{|x-2|}{x-2}$?

a) $f(x)=1$ b) $f(x)=\begin{cases}-1 & \text{if } x<2\\ 1 & \text{if } x\geq 2\end{cases}$ c) $f(x)=\begin{cases}-1 & \text{if } x\leq 2\\ 1 & \text{if } x>2\end{cases}$

d) $f(x)=\begin{cases}-1 & \text{if } x<2\\ 1 & \text{if } x>2\end{cases}$ e) $f(x)=1,\ x\neq 2$

Absolute Bliss (continued)

12) Given $f(x)=\begin{cases}5-3x & \text{if } x<2\\ 10 & \text{if } x=2\\ 3x^2-1 & \text{if } x>2\end{cases}$ which of the following statements are true?

I. $\lim\limits_{x\to 2^+} f(x)=10$ **II.** $\lim\limits_{x\to 2^-} f(x)=-1$ **III.** $\lim\limits_{x\to 2} f(x)=10$

a) I only b) II and III only c) II only d) III only e) I and III only

13) If $g(h(x))=\sqrt{x^2-4}$, $g(x)=\sqrt{x-9}$, and $h(x)>0$ for all real x, then $h(x)=$

a) $\sqrt{(x-9)^2-4}$ b) $\sqrt{x^2-13}$ c) x^2+5 d) $\sqrt{x^2+5}$ e) x^2-13

14) The graph of $f(x)=\begin{cases}-x^2 & \text{if } x\le 0\\ x^3 & \text{if } x>0\end{cases}$ is symmetric with respect to which of the following?

I. The y-axis **II.** The origin **III.** The x-axis

a) I only b) II only c) III only d) I and II only e) None of these

15) The domain of $f(x)=\sqrt{\dfrac{-(x-3)^3}{x-2}}$ is:

a) $[2,3]$ b) $(2,3)$ c) $(2,3]$ d) $(-\infty,2)\cup(3,\infty)$ e) $(-\infty,2)\cup[3,\infty)$

16) The range of $f(x)=\begin{cases}1 & \text{if } 0<x<2\\ -3 & \text{if } x=0\\ -5 & \text{if } x<0\end{cases}$ is:

a) $(-\infty,\infty)$ b) $(0,2)$ c) $[0,2)$ d) $(-\infty,2)$ e) $\{y \mid y=1,\ y=-3,\ y=-5\}$

17) The range of $g(x)=-|x-2|+5$ is:

a) $\{y \mid y\ge 2\}$ b) $\{y \mid y\le 2\}$ c) $\{y \mid y\ge 5\}$ d) $\{y \mid y\le 5\}$ e) $\{y \mid y\in \Re\}$

18) Given $f(x)=\dfrac{|x^3|-1}{x^2}$. How many of the following statements are true?

I. The domain is $(-\infty,0)\cup(0,\infty)$. **II.** The function is odd. **III.** The range is $(-\infty,\infty)$

IV. The function has two zeros. **V.** For large positive values of x the values of $f(x)$ are also large.

a) 1 b) 2 c) 3 d) 4 e) 5

19) Given $f(x)=\begin{cases}3kx^2-2; & x\le -1\\ 2kx^3-5; & x>-1\end{cases}$. Find the value of k such that $\lim\limits_{x\to -1} f(x)$ exists.

a) 3 b) 7 c) -3 d) $-\frac{3}{5}$ e) -7

For the following problems, you are required to write up your answers in a form which includes an explanation of your methods. For example, in problem 20 part **d** *how* did you actually figure the estimate of the instantaneous rate of change of price?

20) Board Prices. If you check the prices of various lengths of lumber you will find that a board twice a long as another of the same type does not necessarily cost twice as much. Let x be the number of feet long a 2×6 board is and let y be the number of cents you pay for the board. Assume that y is given by

$y = 0.2x^3 - 4.8x^2 + 80x$.

a) Find the price of a 2×6 that is 5 feet long, 10 feet long, and 20 feet long.

b) Find the average rate of change of the price, cents per foot, for 5 to 5.1 feet, 5 to 5.01 feet, and 5 to 5.001 feet.

c) The average number of cent per foot in part (b) is approaching an *integer* as the change in x gets smaller and smaller. What integer? What name would you give this rate of change?

d) Estimate the instantaneous rate of change of price if x is 10 feet and if x is 20 feet. You should find that each of these rates is an integer.

e) One of the principles of marketing is that when you buy in larger quantities, you usually pay less per unit. Explain how the numbers in this problem show that this principle does *not* apply to buying longer boards. Why does it not apply?

21) The object of this problem is to determine the effect on the graph of $y = f(x)$ of changing the parameters A, B, C, and/or D in the function $g(x) = A \cdot f(Bx + C) + D$. How does the new function g relate to the original function f when one or more of the parameters A, B. C, or D is changed? In particular, how are the domain, range, and symmetries of a function changed when the function f is changed in g ? How does the shape of the graph of the function change? Most importantly, what effect does each of these parameters have on a function f that causes these changes to occur?

A) Graph each of the following functions, with appropriate domains and ranges. Records the results with paper and pencil, labeling the graphs and including the scales of both axes.

function	$y = f(x)$	f is the parent function of:	Symmetric with respect to: y-axis	origin
1.	$y = x$	$y = mx + b$		
2.	$y = x^2$	$y = ax^2 + bx + c$		
3.	$y = \|x\|$	$y = a\|bx + c\| + d$		
4.	$y = \sin x$	$y = a\sin(bx + c) + d$		
5.	$y = \frac{1}{x}$	$y = \frac{a}{(bx + c)} + d$		

B) Using the functions 3, 4 and 5 in part (a), investigate the effect of adding a constant to the function $y = f(x)$.

a) Graph the following: i. $f(x) = |x|$ ii. $g(x) = |x| + 3$ iii. $g(x) = |x| - 4$

Graph $f(x) = |x|$ on $[-10,10]$ by $[-6, 6]$. Overlay the graph $g(x) = |x| + 3$. (What is D?) Overlay the graph $g(x) = |x| - 4$ on the same viewing rectangle (D = – 4 this time).

b) Repeat part (a) for functions 4 and 5, graphing $y = f(x)$, $g(x) = f(x) + 3$, and $g(x) = f(x) - 4$ in each case. Clear the graphics screen before graphing each new set of related functions.

c) How does the change in D effect the graph of f?
Describe the shape and position of the graph considering D.

C) Using functions 3, 4, and 5 from part A, investigate the effect of changing the parameter C in the function

$g(x) = A f(B x + C) + D.$

a) For each of the functions 3 and 5 in part (A) compare the graphs of the following functions. Using an appropriate domain and range, sketch all three functions on the same viewing rectangle.

i. $y = f(x)$ ii. $g(x) = f(x + 2)$ iii. $g(x) = f(x - 4)$

b) Using function 4, graph $y = f(x + \pi)$, $y = f\left(x + \frac{\pi}{2}\right)$, $y = f\left(x - \frac{\pi}{2}\right)$ together on the viewing rectangle [– 6.3, 6.3] by [– 1.5, 1.5]. What familiar trigonometric functions (if any) do these graphs resemble?

c) Considering the results from observations made in parts a and b, describe the shape and position of the graph $g(x) = f(x + C)$ when: i. $C > 0$ Shifts graph Left ii. $C < 0$ Shifts graph Right

d) Why does the graph of g appear as it does as compared to the graph of f. Describe the effect of first adding C to x then performing the function on this quantity. moved to Right of f

22) Graph $f(x) = \dfrac{x^2 - 3x - 4}{x^2 - 1}$ on [– 2, 2] by [– 10, 10] by putting $f(x)$ into Y_1. Note that $y = f(x)$ undefined at $x = -1$ and $x = 1$.

A) From the graph, how does f appear to behave near $x = -1$? near $x = 1$?

B) For values of x (1 or – 1) where the graph appears to be continuous (has no visib le holes), use the trace option of the graphics utility to estimate the valy of y for this choice of x.

C) To determine the behavior of $f(x) = \dfrac{x^2 - 3x - 4}{x^2 - 1}$ for x near 1 numerically, complete the following table of values. Use the program BEHAVIOR. To complete the first two columns, let H = .5. For the last two columns, let H = – .5 In this part use A = 1. (In part F below use A = – 1.) Notice that for each successive table value, the value of H is half that of the previous entry for H.

```
Program; New; <Enter>
BEHAVIOR  <Enter>
Prompt A
Prompt H
Lbl 1
A + H store X
Disp "X"
Disp X
Disp "Y1"
Disp Y1
Pause
H/2 store H
Goto 1
```

H	A + H	f(A + H)	A – H	f(A – H)
0.5	1.5	– 5	0.5	7
0.25	1.25	– 11	0.75	13
0.125		-23	0.875	25
0.0625		-47	0.9375	49
0.03125		-95		97
0.015625		-191		193
0.0078125		-383		385
0.00390625		-767		769
0.001953125		-1535		1537

D) Observe the magnitude of successive values of y in the table above. Describe the behavior of $f(x) = \dfrac{x^2 - 3x - 4}{x^2 - 1}$ for x near 1.

E) Compare the values in the table to the graph of . How are these related? Explain.

F) To determine the behavior of $y = f(x)$ for values of x near – 1, complete the table below.

G) Compare the successive y–values listed in part F to graph of f. Explain how these are related.

H	A + H	f(A + H)	A – H	f(A – H)
0.5	– 0.5	3	– 1.5	2.2
0.25	– 0.75	2.714285714	– 1.25	2.3333333333
0.125		2.6	– 1.125	2.4117
0.0625		2.548	– 1.0625	2.45
0.03125		2.524		2.48
0.015625		2.511		2.49
0.0078125		2.505		2.495
0.00390625		2.503		2.50
0.001953125		2.501		2.50

(7) $ax+by=3$ $bx+ay=2$ $(4,-1)$

$4a-b=3$ $4b-a=2$

$b=4a-3$

(B)

$4(4a-3)-a=2$

$16a-12-a=2$

$15a=14$

$a=\frac{14}{15}$

(8)

$f(x)=x^2+3x$ $g(x)=x^3-x$ $f(g(2))$

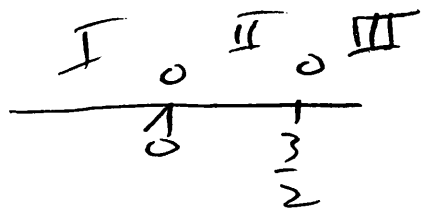
D $f(6)=36+18$

54

$8-2$

6

(9) $f(x)=|3x-2|+2|x|-5$

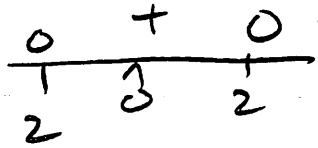


wants II $-(3x-2)+2x-5$

$-3x+2+2x-5$

(A) $-x-3$

(10) $f(x)$ $\frac{3x}{\sqrt{4-x^2}}$ $\sqrt{(2-x)(2+x)}$



$|x|<2$

(B)

(11) $f(x)=\frac{|x-2|}{x-2}$ D NOT B because cannot include $x=2$.

Ab Calc U

(12) $f(x)=\begin{cases} 5-3x & \text{if } x<2 \\ 10 & \text{if } x=2 \\ 3x^2-1 & \text{if } x>2 \end{cases}$

(C)

I $\lim_{x\to 2^+} f(x)=10$ II $\lim_{x\to 2^-} f(x)=-1$ III $\lim_{x\to 2} f(x)=10$

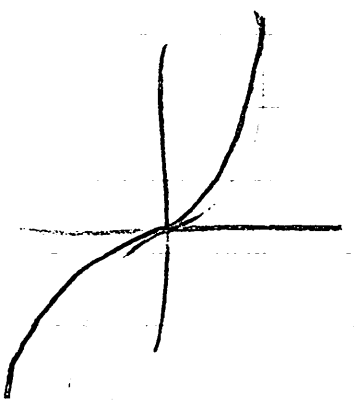
NO = 11 YES NO

(L) ≠ (R)

(13) $g(h(x))=\sqrt{x^2-4}$ $g(x)=\sqrt{x-9}$

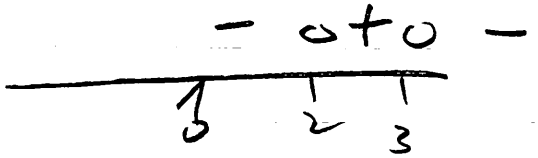
(4) $h=(x^2+5)$

(14) $f(x)=\begin{cases} -x^2 & \text{if } x\le 0 \\ x^3 & \text{if } x>0 \end{cases}$



(E) NOT Symmetric at all

(15) $f(x)=\sqrt{\dfrac{-(x-3)^3}{x-2}}$



(C) D_f $-2<x\le 3$ C

(16) $f(x)=\begin{cases} 1 & \text{if } 0<x<2 \\ -3 & \text{if } x=0 \\ -5 & \text{if } x<0 \end{cases}$ E

(E)

(17) $g(x) = -|x-2|+5$

R_g $g(x) \leq 5$

(D)

(18) $f(x) = \dfrac{|x^3|-1}{x^2} \rightarrow$ if $x>0$ $x - \frac{1}{x^2}$; if $x<0$ $-x - \frac{1}{x^2}$

I True D_f $x \neq 0$

II No fctn is neither odd nor even

III Range $\neq \mathbb{R}$? Yes.

(A) ± 1

IV 2 zeros

V $\lim_{x \to \pm\infty} f(x) \to \infty$

Yes

at $x = \pm 1$

Yes,

$\therefore$ D

(19) $f(x) = \begin{cases} 3kx^2 - 2 & x \leq -1 \\ 2kx^3 - 5 & x > -1 \end{cases}$

$\lim_{x \to 1^-} f(x) = 3k-2$

$\lim_{x \to 1^+} f(x) = -2k - 5 \overset{\text{set}}{=} 3k-2$

$-3 = 5k$

$-\frac{3}{5} = k$

(D)

Abs Rtns VI

A) $D_A \; x \neq \{-2, -1, 3\}$

B) $D_B \; x \neq \{3, \frac{5}{2}, 2\}$

$\frac{1}{x-3} \overset{Set}{=} -2$

$1 = -2x + 6$

$2x = 5$

$x = \frac{5}{2}$

$\frac{1}{x-3} \overset{Set}{=} -1$

$1 = -x + 3$

$x = 2$

C) $\frac{1}{x-3} \overset{Set}{=} 3$ $D_C \; x \neq \{3, \frac{10}{3}\}$

$1 = 3x - 9$

$3x = 10$

$x = \frac{10}{3}$

D) $- \; o \; +$ (0, 3)

$D_D \; x > 3$

E) $- \; o \; + \; o \; -$ (-3, -2, -1)

$D_E \; -3 \leq x < -2$ or $x > -1$

F) $D_F \; x \neq \{-2, -1, 3\}$

G) $+ \; o \; - \; o \; +$ (-3, -2, -1)

$D_G \; -3 < x < -2$ or $x > -1$

H) $D_H \; x \neq \{-2, -1\}$

$\frac{x+3}{x^2+3x+2} \overset{Set}{=} -2$

$x + 3 = -2x^2 - 6x - 4$

$2x^2 + 7x + 7 = 0$

$x = \frac{-7 \pm \sqrt{49 - 56}}{4}$ reject.

$\frac{x+3}{x^2+3x+2} \overset{Set}{=} -1$

$x + 3 = -x^2 - 3x - 2$

$x^2 + 4x + 5 = 0$

$x = \frac{-4 \pm \sqrt{16 - 20}}{2}$ reject.

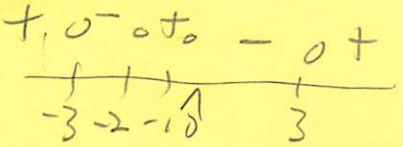
No further restrictions.

I) $+ \; o$ (3)

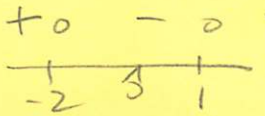
$D_I \; x \geq 3$

J) $o \; +$ (3)

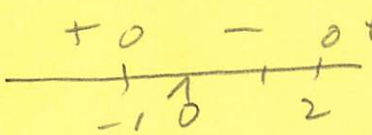
$D_J \; x > 3$

K) 

D_K $x < -3$ or $-2 < x < -1$ or $x > 3$

L) 

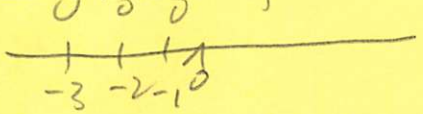
D_L $x \leq -2$ or $x \geq 1$

M) 

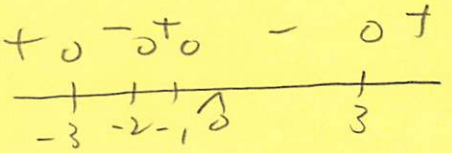
D_M $x \leq -1$ or $x \geq 2$

N)

D_N $x < -2$ or $x > 1$

O) 

D_O $-3 < x < -2$ or $x > -1$

P) 

D_P $x < -3$ or $-2 < x < -1$ or $x > 3$

Q) $x+5 \overset{\text{Sub}}{=} -2$

$x = -7$

$x+5 \overset{\text{Sub}}{=} -1$

$x = -6$

D_Q $x \neq \{-7, -6, -5\}$

R) $x^2+x-2 \overset{\text{Sub}}{=} -5$

$x^2+x+3 = 0$

$x = \dfrac{-1 \pm \sqrt{1-12}}{2}$

Reject.

D_R $x \in \mathbb{R}$

S) D_S $x \neq \{-5, -10\}$.

$x+5 \overset{\text{Sub}}{=} -5$

$x = -10$

t) D_t $x \neq \{5, 0\}$

$x - 5 \overset{\text{set}}{=} -5$

$x = 0$

u) $x - 5 \overset{\text{set}}{=} -5$

$x = 0$

D_u $x \neq \{-5, 0\}$

v) $x + 5 \overset{\text{set}}{=} -5$

$x = -10$

D_v $x \neq \{-5, -10\}$

x) $x \neq -5$

w) D_w $x \neq \left\{3, \frac{10}{3}, \frac{-17}{5}, \frac{-7}{2}\right\}$

$\frac{1}{x-3} \overset{\text{set}}{=} 3$

$1 = 3x - 9$

$3x = 10$

$x = \frac{10}{3}$

d(c) →

$\frac{1}{x-3} \overset{\text{set}}{=} -2$

$1 = -2x + 6$

$2x = 5$

$x = \frac{5}{2}$

what does x have to be to make $\frac{5}{2}$ which will cause the domain problem

$\frac{1}{x-3} \overset{\text{set}}{=} \frac{5}{2}$

$2 = 5x - 15$

$5x = -17$

$x = \frac{-17}{5}$

$\frac{1}{x-3} \overset{\text{set}}{=} -1$

$1 = -x + 3$

$x = 2$

$\frac{1}{x-3} \overset{\text{set}}{=} 2$

$1 = 2x - 6$

$2x = -7$

$x = \frac{-7}{2}$

9. Speed $\Rightarrow 3t^2$ a) $3\left(\frac{1}{2}\right)^2 = \frac{3}{4}$ b) 3 c) 12

10. $c^4 - d^4 = (c^3 + c^2d + cd^2 + d^3)(c-d)$

$(c^2 - d^2)(c^2 + d^2) = c^2(c+d) + d^2(c+d)(c-d)$

$(c-d)(c+d)(c^2+d^2) = (c+d)(c^2+d^2)(c-d)$

11. $\frac{(t_1)^4 - (t)^4}{t_1 - t} = t_1^3 + t_1^2 t + t_1 t^2 + t^3$ as $t_1 \Rightarrow t \Rightarrow 4t^3$

12.

$$c - d \;\overline{)\; c^5 - d^5 \;\;\; c^4 + c^3d + c^2d^2 + cd^3 + d^4}$$

$c^5 - c^4d$

c^4d

$c^4d - c^3d^2$

c^3d^2

$c^3d^2 - c^2d^3$

$c^2d^3 - cd^4$

cd^4

$\frac{t_1^5 - t^5}{t_1 - t} \Rightarrow 5t^4$

13. A) $\frac{\sqrt{4.1} - \sqrt{4}}{.1} = \frac{2.02 - 2}{.1} = \frac{.02}{.1} = .2$

B) $\frac{\sqrt{t} - \sqrt{4}}{t - 4} = \frac{1}{\sqrt{t} + \sqrt{4}} \Rightarrow$ as $t \to 4$ $\frac{1}{4}$

14. A) approaches 1 B) $\frac{\left(1 - \frac{1}{t}\right) - \frac{2}{3}}{t - 3} = \frac{\frac{1}{3} - \frac{1}{t}}{t-3} = \frac{\frac{t-3}{3t}}{t-3} = \frac{1}{3t}$, as $t \to 3$ speed is $\frac{1}{9}$

c) $\frac{1 - \frac{1}{t_1} - \left(1 - \frac{1}{t}\right)}{t_1 - t} = \frac{\frac{1}{t} - \frac{1}{t_1}}{t_1 - t} = \frac{1}{tt_1} \Rightarrow \frac{1}{t^2}$

diff of squares

diff of sq.

1st

A) $\frac{(2.1)^2 - 2^2}{2.1 - 2} = \frac{4.41 - 4}{.1} = 4.1$

B) $\frac{x_1^2 - 2^2}{x_1 - 2} = x_1 + 2$

C) $2 + 2 = 4$

① object travels at least 40 feet and at most 50 feet.

② at least at most

$0-1/3$	0	$\frac{1}{27}$
$1/3-2/3$	$\frac{1}{27}$	$\frac{4}{27}$
$\frac{2}{3}-1$	$\frac{4}{27}$	$\frac{9}{27}$
$1-1\frac{1}{3}$	$\frac{9}{27}$	$\frac{16}{27}$
$1\frac{1}{3}-1\frac{2}{3}$	$\frac{16}{27}$	$\frac{25}{27}$
$1\frac{2}{3}-2$	$\frac{25}{27}$	$\frac{36}{27}$
$2-2\frac{1}{3}$	$\frac{36}{27}$	$\frac{49}{27}$
$2\frac{1}{3}-2\frac{2}{3}$	$\frac{49}{27}$	$\frac{64}{27}$
$2\frac{2}{3}-3$	$\frac{64}{27}$	$\frac{81}{27}$

$\frac{204}{27}$

7.55^{-}

at least

$\frac{285}{27} = 10.55^{-}$

at most.

$N = L^2$

$\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2$

③ A)

	Least	most
$0-1$	0	1
$1-2$	1	4

at least 1 at most 5

B)

$0-1/2$	0	$\frac{1}{8}$
$\frac{1}{2}$ 1	$\frac{1}{8}$	$\frac{1}{2}$ $\frac{9}{8}$
1 $\frac{3}{2}$	$1/2$	
$\frac{3}{2}-2$	$\frac{9}{8}$	2

at least $\frac{7}{4}$ at most $\frac{15}{4}$

$(L^2)(\text{length of time})$

C)

$0-\frac{1}{3}$	0	$\frac{1}{27}$
$\frac{1}{3}-\frac{2}{3}$	$\frac{1}{27}$	$\frac{4}{27}$
$\frac{2}{3}-1$	$\frac{4}{27}$	$\frac{9}{27}$
$1-1\frac{1}{3}$	$\frac{9}{27}$	$\frac{16}{27}$
$1\frac{1}{3}-1\frac{2}{3}$	$\frac{16}{27}$	$\frac{25}{27}$
$1\frac{2}{3}-2$	$\frac{25}{27}$	$\frac{36}{27}$

at least $\frac{55}{27}$ at most $\frac{91}{27}$

④ $\int \sqrt{t}\,dt$ / Sec at time $= t$.

A) 1st $\frac{1}{2}$ Sec $0 - \frac{1}{2}$ 0 $\frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$ rect.

B) $\frac{1}{2} - 1$ $\frac{1}{4}$ $1\cdot\frac{1}{2} = \frac{1}{2}$ rect

C) $\frac{1}{4}$ Cost $\frac{3}{4}$ rect

⑤

0	$\frac{1}{4}$	0	$\frac{1}{16}$
$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{16}$	$\frac{2}{16}$
$\frac{1}{2}$	$\frac{3}{4}$	$\frac{2}{16}$	$\frac{3}{16}$
$\frac{3}{4}$	$\frac{4}{4}$	$\frac{3}{16}$	$\frac{4}{16}$

at least $\frac{3}{8}$ at most $\frac{5}{8}$

⑥ Generalize. 10 Sections each $\frac{1}{10}$

$\sum_{j=1}^{N} j = \frac{N}{2}(N+1)$

0	$\frac{1}{100}$
	$\frac{2}{100}$
	⋮
$\frac{9}{100}$	$\frac{10}{100}$

$\frac{45}{100}$ at least $\frac{55}{100}$ at most.

⑦ $\int \sqrt{t}$ From $t=1$ to $t=4$

A)

$1-2$	1.000	1.414
$2-3$	1.414	1.732
$3-4$	1.732	2.000
	4.146	5.146

B)

1	$\frac{3}{2}$	1	1.225
$\frac{3}{2}$	2	1.225	1.414
2	$\frac{5}{2}$	1.414	1.581
$\frac{5}{2}$	3	1.581	1.732
3	$\frac{7}{2}$	1.732	1.871
$\frac{7}{2}$	4	1.871	2.000

each entry needs to be multiplied by time $\frac{1}{2}$

at most 4.911

at least 4.4115

(8)

A) $2-3$ $.5$ $.3\overline{3}$ at least & 8
$3-4$ $.3\overline{3}$ $.25$ at most & 3

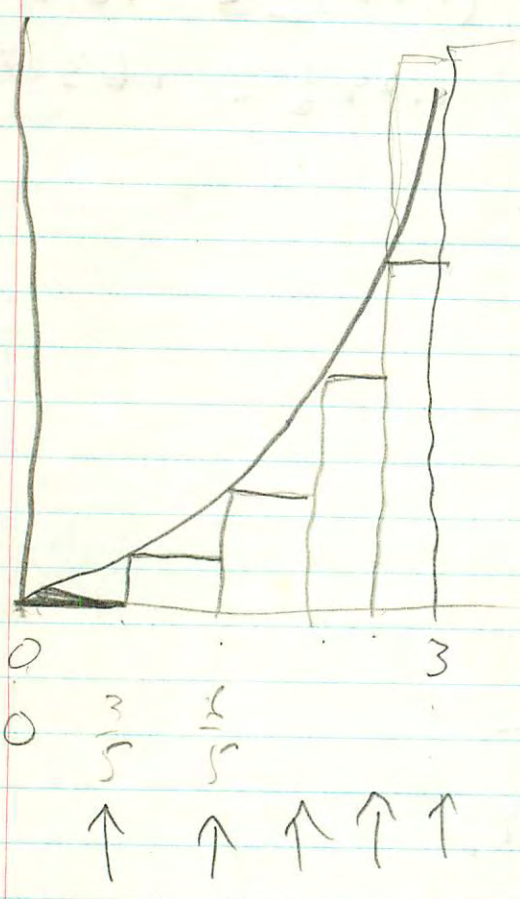
B)

2	$\frac{5}{2}$	.5	.40
$\frac{5}{2}$	3	.4	.333
3	$\frac{7}{2}$	.333	.29
$\frac{7}{2}$	4	.29	.25
		1.52	1.29

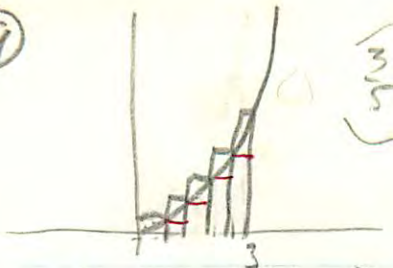
$(1.\overline{52})\frac{1}{2} = .76$ at most
$1.29\frac{1}{2} = .63$ at least

C)

2	$2\frac{1}{3}$	.167	.143
$2\frac{1}{3}$	$2\frac{2}{3}$	.143	.125
$2\frac{2}{3}$	3	.125	.111
3	$3\frac{1}{3}$	.111	.100
$3\frac{1}{3}$	$3\frac{2}{3}$	.100	.091
$3\frac{2}{3}$	4	.091	.083
		.737	.673
		at most	at least


0
3
0
3/5
4/5

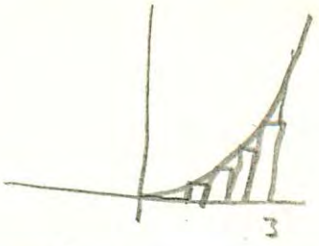
(9)



(9)

$\frac{3}{5}$ $\frac{6}{5}$ $\frac{9}{5}$ $\frac{12}{5}$ $\frac{15}{5}$

(10)



10

length of interval $\frac{3}{5}$

height $(L)^2$

1st $\frac{9}{25} \cdot \frac{3}{5} = \frac{27}{125}$

2nd $\frac{36}{25} \frac{3}{5} = \frac{108}{125}$

3rd $\frac{81}{25} \frac{3}{5} = \frac{243}{125}$

4th $\frac{144}{25} \cdot \frac{3}{5} = \frac{432}{125}$

5th $\frac{225}{25} \cdot \frac{3}{5} = \frac{675}{125}$

$\frac{1485}{125} = 11.88$

So area under curve is less than 11.88

interval length is still $\frac{3}{5}$ because Red lines on (9) is figure for (10)

$\frac{810}{125} = 6.48$

So area under curve is more than 6.48.

(11) (9) is upper bound
(10) is lower bound. on dist traveled by Rocket for the time period ab 3 sec.

$$\frac{11.88 + 6.48}{2} = \frac{18.36}{2} = 9.18$$

if If we were to approx dist for rocket & used the same interval the computations would be the same, we know Rocket travels t^2 ft/sec. so at 3 sec it is traveling at 9 ft/sec however at $t=3$ sec the rocket will have traveled 9 feet since $t=0$. This is what the area under the curve $y=x^2$ (which is the rocket's speed) means.

GUIDE Quiz on Chapt 1

① (A) $(1.2)^3 - 1. _ = 1.728 - 1. _ = .728$

(B) $\frac{.728}{1.2-1} = \frac{.728}{.2} = 3.64$

② (Speed)(Time interval)

	at left	at mid.
0 – .2	6.0	7.6
.2 – .4	7.6	8.8
.4 – .6	8.8	9.8
.6 – .8	9.8	10.4
.8 – 1.0	10.4	10.8
	42.6	47.4

In Level. 2

③ $\frac{2\tau_1^3 + 5\tau_1 - (2\tau^3 + 5\tau)}{\tau_1 - \tau} \quad \frac{2(\tau_1^3 - \tau^3) + 5(\tau_1 - \tau)}{\tau_1 - \tau} =$

$\frac{(\tau_1 - \tau)(2(\tau_1^2 + \tau_1\tau + \tau^2) + 5)}{\tau_1 - \tau} = 2\tau_1^2 + 2\tau_1\tau + 2\tau^2 + 5 \Big]_{\tau=2}$

$8 + 8 + 8 + 5 = 29$ ft/sec,

$\frac{3}{2}$

④

A) $1 - 1\frac{1}{2}$ $\quad 1$ $\quad \frac{27}{16}$

$1\frac{1}{2} - 2$ $\quad \frac{27}{16}$ $\quad \frac{64}{16}$

Least $\frac{43}{16}$ $\quad \frac{71}{16}$ ave 3.56

(B) $1 - 1\frac{1}{3}$ $\quad \frac{1}{3}$ $\quad \frac{64}{81}$

$1\frac{1}{3} - 1\frac{2}{3}$ $\quad \frac{64}{81}$ $\quad \frac{125}{81}$

$1\frac{2}{3} - 2$ $\quad \frac{125}{81}$ $\quad \frac{216}{81}$

$\frac{405}{81} = \frac{45}{9} = 5$

$\frac{190}{81} = 2.346$

3.673

2nd Day of School

Pg 15 #4 $f(x) = x^3$

a) compute $f(3.01)$ A

b) $f(3)$ B

c) $f(3.01) - f(3)$ C

d) $\dfrac{f(3.01) - f(3)}{0.01}$ ~~D~~

1 turn on

2 ~~Y= clear Zoom 6~~ use arrow keys to get there

3. Y=

4. clear X,T,θ to get variable

5. ~~x² log;~~ MATH choose #3

6. 2nd quit. to get to home screen.

7. Zoom 6 Graph (for fun).

8. Zoom 2 ENTER more fun.

9. ~~2nd~~ Go to Homescreen again

10. 2nd Y-VARS ENTER ENTER ~~STO ALPHA~~ A

11. (3.01) ENTER 27.270901 STO ALPHA A

12 ~~X~~ 2. 2nd ENTER to get Last entry. 2 times.

13 ~~X~~ 3. Delete to get Y1(3) ENTER 27. ≡

14 ~~4.~~ ~~2nd~~ (ANS) STO ALPHA B ENTER

15. ALPHA A — ALPHA B STO ALPHA C ENTER

16. ALPHA C ÷ .01 ENTER 27.0901

Alg Review

① $2x(y+3x) = 2xy+6x^2$ C

② $(a+b)^2 = a^2+2ab+b^2$ D

③ $(3+x)(5+y) = 15+3y+5x+xy$ C

④ $x^3-3x=0 \quad x \in \{0, \pm\sqrt{3}\}$ B

⑤ $\sqrt{2}-1$ a $\Rightarrow x=\pm\sqrt{3}$ b $\frac{2\pm\sqrt{4+4}}{2}$ C $\frac{-2\pm\sqrt{4+4}}{2}$ C

$\frac{2\pm 2\sqrt{2}}{2}$ $\frac{-2\pm 2\sqrt{2}}{2}$

$1\pm\sqrt{2}$ $-1\pm\sqrt{2}$

⑥ $\frac{1}{x} = \frac{1}{a}+\frac{1}{b}$

$\frac{1}{x} = \frac{a+b}{ab}$

$x = \frac{ab}{a+b}$ C

⑦ $y=3x-1$ $y=\frac{3}{2}-1$

$y=5x-2$ $y=\frac{1}{2}$

$0=-2x+1$ C

$x=\frac{1}{2}$

⑧ $-4[7-3(6-9)] = -4[7+9] = -64$ D

⑨ $\sqrt{20}+\sqrt{45} = 2\sqrt{5}+3\sqrt{5} = 5\sqrt{5}$ D

⑩ $(2x+1)(x-1)=1$ ⑪ D ⑫ B ⑬ $\frac{1}{a+b}+\frac{1}{a-b}$ C

$2x^2-x-2=0$ $\frac{a-b+a+b}{a^2-b^2} = \frac{2a}{a^2-b^2}$

$x=\frac{1\pm\sqrt{1+16}}{4}$ ⑭ $\sqrt{x^2+9x^4} =$

B $x\sqrt{1+9x^2}$ B

⑮ $x^3-3x\Big]_{x=-1}$ $-1+3=2$ D

(1) a) 16 B) 16 c) 2 D) 0

(2) a) 2 B) 8 c) $\frac{1}{8}$

(3) a) $\mathbb{R}$ B) $\mathbb{R}$ c) ≥ 0 D) $\mathbb{R}, x \neq 0$ E) $\mathbb{R}$ F) $\mathbb{R}$ $x \notin \{0, -1\}$

(4) a) 9.0601 B) 9 c) .0601 D) 6.01

(5) a) 5 B) 6.05 c) 1.05 D) 10.5

(6) a) $-.08\overline{3}$ B) $-.108$ c) $-.115$

(7) a) .236 B) .248 c) .25

$$x^2+2xh+h^2$$
$$x+h$$
$$x^3+2x^2h+xh^2$$
$$x^2h+2xh^2+h^3$$

(8)

a) $\frac{(x+h)^2 - x^2}{h} = \frac{x^2+2xh+h^2-x^2}{h} = 2x+h$

B) $\frac{(x+h)^3 - x^3}{h} = \frac{x^3+3x^2h+3xh^2+h^3}{h} = 3x^2+3xh+h^2$

c) $\frac{2(x+h)-2x}{h} = \frac{2x+2h-2x}{h} = 2$

D) $\frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \frac{x-x-h}{hx(x+h)} = \frac{-1}{x(x+h)}$

(9) a) .2 b) $-.5$ c) 10 D) .4 E) .5 F) $-.2$

(10) a) 4.5 B) 15.25 c) $-.2$

(11) a) 7 B) x^2-9 $(x-3)(x+3)$ c) .61 D) 12

(12) a) 4.2025 B) 3.61

(13) a) .25 B) 1 c) .2 D) $-.05$

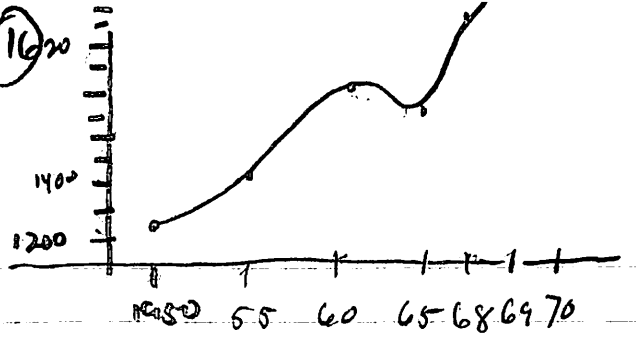
(14) a) $\frac{1}{3} - \frac{1}{2} = \frac{-1}{6}$ B) $.05\overline{3}$ c) $\frac{1}{t} - \frac{1}{2} = \frac{2-t}{2t}$

D) $\frac{1}{x} - \frac{1}{2} = \frac{2-x}{2x}$ E) $-.4$ F) $.08\overline{3}$ g) $\frac{h}{24}$

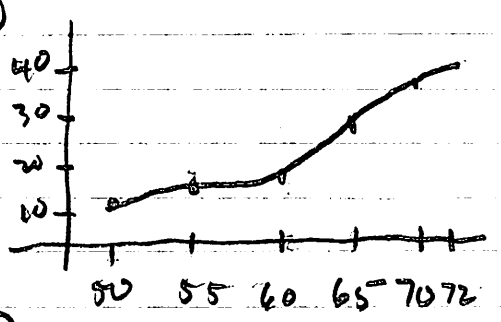
(15)

a) $\frac{2.07-2}{.3} = .23\overline{3}$ B) $\frac{2.05-2}{.2} = .25$ c) $\frac{2.02-2}{.1} = .2$

(16)



(17)



(18)

a) Average speed traveled for $t_1 - t$ seconds.

B) average acceleration for $t_1 - t$ seconds.

(19) $\dfrac{t_1^2 - t^2}{t_1 - t} = t_1 + t$

(20) $\dfrac{5t_1 - 5t}{t_1 - t} = \dfrac{5(t_1 - t)}{(t_1 - t)} = 5$

(21) a) $f(1) = 1^2 = 1 = (-1)^2 = f(-1)$

b) $f(f(3)) = f(9) = 81$

c) $f(x^2) = (x^2)^2 = x^4$

(22) $f(x) = \dfrac{x}{x-1}$

a) $x \in \mathbb{R}\ x \neq 1$

B) 2

c) $+\frac{1}{2}$

D) 2

E) 3

f) $f\left(\dfrac{x}{x-1}\right) = \dfrac{\frac{x}{x-1}}{\frac{x}{x-1} - 1} = \dfrac{\frac{x}{x-1}}{\frac{x-(x-1)}{x-1}} = x$

(23) $f(x) = \dfrac{1}{1-x}$

a) $x \in \mathbb{R}\ x \neq 1$

b) $-1, \frac{1}{2}, 2$

$f(f(x)) = f\left(\dfrac{1}{1-x}\right) = \dfrac{1}{1 - \frac{1}{1-x}} = \dfrac{1}{\frac{1-x-1}{1-x}} = \dfrac{1-x}{-x} = \dfrac{x-1}{x}$

$f(f(f(x))) = f\left(\dfrac{x-1}{x}\right) = \dfrac{1}{1 - \frac{x-1}{x}} = \dfrac{1}{\frac{x-(x-1)}{x}} = \dfrac{1}{\frac{1}{x}} = x$

$f(a+b) = f(a) + f(b)$

A $f(x) = 3x$

1st $f(a) = 3a$

2nd $f(b) = 3b$

3rd $f(a+b) = 3(a+b)$

4th are they equal? Yes

(B) $f(x) = 3x+2$

$f(x+y) = 3(x+x)+2 \stackrel{?}{=} f(x)+f(x)$

$6x+2 \stackrel{?}{=} 3x+2+3x+2$

$6x+2 \neq 6x+4$ NO

C) $f(x) = x^2$

$f(x+x) = (x+x)^2 \stackrel{?}{=} f(x)+f(x)$

$= 4x^2 \stackrel{?}{=} x^2+x^2$

$\neq 2x^2$ NO

D) $f(x) = -x$

$f(x+x) = -(x+x)$

$= -2x = -2x$ Yes,

E) $f(x) = x+4$ $f(a+b) = a+b+4$

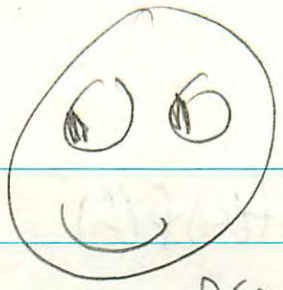
$f(a) = a+4$

$f(b) = b+4$ NO

F) $f(x) = |x|$ $f(a)+f(b) = f(a+b)$

$|a|+|4| = |a+4|$ NO

#24 $f(a+b) = f(a) + f(b)$



A) $f(x) = 3x$.

$f(2x) = f(x+x) \stackrel{?}{=} f(x) + f(x)$

$f(2x) = 3x + 3x$

$3(2x) \rightarrow 6x = 6x$ Yes.

$f(0+x) = f(0) + f(x)$

$f(x)$

$3x = 3(0) + 3x$

$3x = 3x$

Yes.

B) $f(x) = 3x+2$

$f(x+x) = f(x) + f(x)$

$f(2x) \stackrel{?}{=} 3x+2+3x+2$

$3(2x)+2 =$

$6x+2 = 6x+4$ NO.

$f(0+x) = f(0) + f(x)$

$3x+2 = 2 + 3x+2$

NO

C) $f(x) = x^2$

$f(x+x) \stackrel{?}{=} f(x) + f(x)$

$f(2x) \stackrel{?}{=} x^2 + x^2$

$(2x)^2$

$4x^2 = 2x^2$ NO

E) $f(x) = \frac{1}{x}$

$f(x+x) = f(x) + f(x)$

$f(2x) = \frac{1}{x} + \frac{1}{x}$

$\frac{1}{2x} = \frac{2}{x}$

NO.

D) $f(x) = -x$

$f(x+x) = f(x) + f(x)$ Yes

$f(2x) \quad -x - x$

$-(2x) =$

$-2x = -2x$

$f(ab) = f(a)f(b)$

A) $f(x) = x^2$

$f(a) = a^2$

$f(b) = b^2$

$f(ab) = a^2b^2 \stackrel{?}{=} f(a) \cdot f(b)$

$a^2 b^2$ Yes

B) $f(x) = \frac{1}{x}$

$f(a) = \frac{1}{a}$ $f(b) = \frac{1}{b}$

$f(ab) = \frac{1}{ab} \stackrel{?}{=} \frac{1}{a} \cdot \frac{1}{b}$ Yes,

C) $f(x) = 3x$

$f(a) = 3a$ $f(b) = 3b$

$f(ab) = 3ab \stackrel{?}{=} 3a \cdot 3b$

$= 9ab$ NO

D) $f(x) = \sqrt{x}$

$f(a) = \sqrt{a}$

$f(b) = \sqrt{b}$

$f(ab) = \sqrt{ab} \stackrel{?}{=} \sqrt{a}\sqrt{b}$ Yes

E) $f(x) = x + 4$ $f(ab) = ab + 4$

$f(a) = a + 4$ NO

$f(b) = b + 4$

F) $f(x) = |x|$

$f(a) = |a|$

$f(b) = |b|$

$f(ab) = |ab|$ Yes

25 $f(ab) = f(a)f(b)$

A) $f(x) = x^2$

$f(x \cdot x) = f(x)(f(x)$

$f(x^2) = x^2 \cdot x^2$

$(x^2)^2 = x^4$ Yes.

B) $f(x) = \frac{1}{x}$

$f(x \cdot x) = f(x) \cdot f(x)$

$f(x^2) = \frac{1}{x} \cdot \frac{1}{x}$

$\frac{1}{x^2} = \frac{1}{x^2}$ Yes

C) $f(x) = 3x$

$f(x \cdot x) = f(x) f(x)$

$f(x^2) = 3x \cdot 3x$

$3x^2 = 9x^2$

NO

D) $f(x) = \sqrt{x}$

$f(x \cdot x) = f(x) f(x)$

$f(x^2) = \sqrt{x} \cdot \sqrt{x}$

$\sqrt{x^2} = x$

$x = x$ if $x \geq 0$

E) $f(x) = x + 4$

$f(x \cdot x) = f(x) \cdot f(x)$

$f(x^2) = (x+4)(x+4)$

$x^2 + 4 =$

NO

34

35

$3(a+b)$

NO.

F) $f(x) = |x|$

$f(x \cdot x) = f(x) \cdot f(x)$

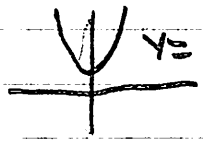
$|x^2| = |x| \cdot |x|$

$x^2 = x^2$

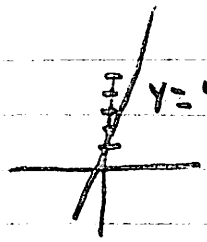
$f(x) = x(x-1)$

1–7 obvious.

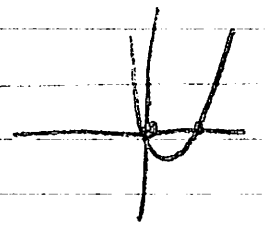
(8) $y = x^2 + 1$



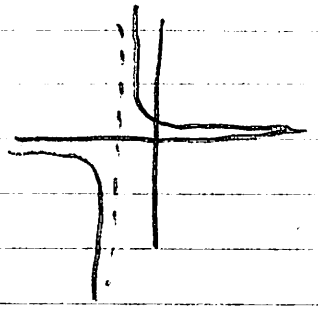
(9) $y = 4x + 1$



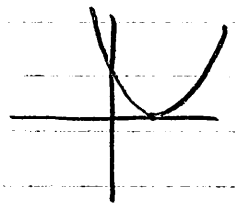
(10)



(11) $f(x) = \frac{1}{1+x}$



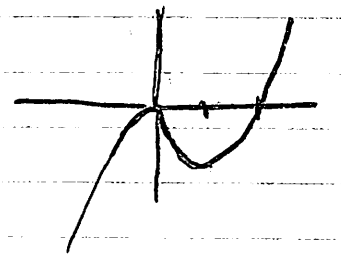
(12) $f(x) = \frac{1}{(x-1)^2}$



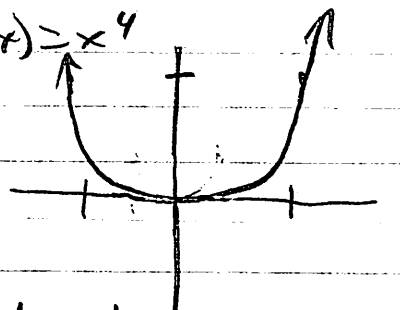
(13) $x^3 - 2x^2 = 0$

$x^2(x-2) = 0$

$x = 0, \; x = 2$



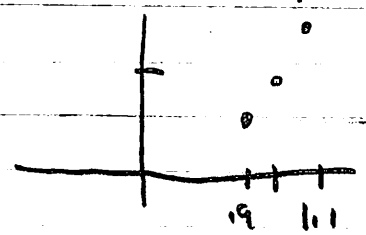
(14) $f(x) = x^4$



almost flat from $+1$ to -1

(15)

x	1	1.1	0.9
x^2	1	1.21	.81



almost ST Line,

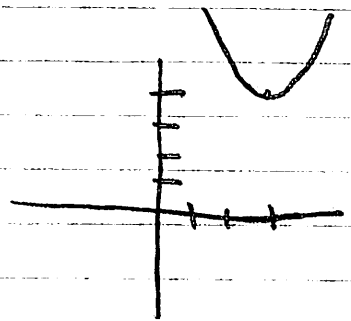
(16) $f(x) = x(x+1)(x-1)$

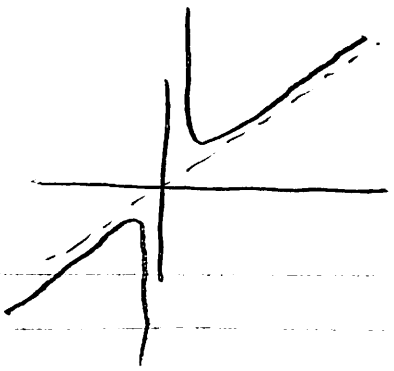
a) $0, 1, -1$

b) at those pts i.e. $(0,0), (1,0), \& (-1,0)$

c) $(0,0)$ only.

(17) $y = (x-3)^2 + 4$





(18) $f(x) = x + \frac{1}{x}$ a) $x \in \mathbb{R}\ x \neq 0$

(19) a) complete the sq.

b) $c = \frac{b^2}{4a}$

c) $ax^2 + bx + c = 0$

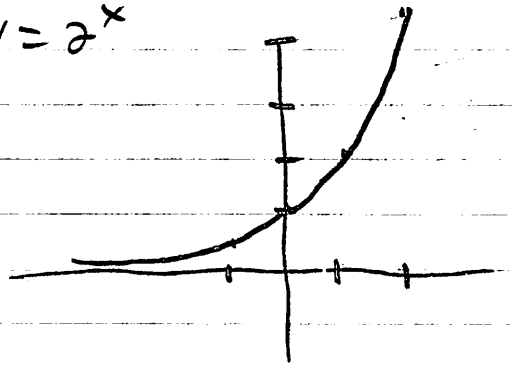
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{1}{2}\left(-b \pm \sqrt{b^2 - 4ac}\right)$$

$$\frac{1}{2}\left(\frac{-b + \sqrt{b^2 - 4ac}}{2a} + \frac{-b - \sqrt{b^2 - 4ac}}{2a}\right) = \frac{-2b}{4a} = -\frac{b}{2a}$$

(20)

a) disc > 0 2 b) None c) 1

(21) $y = 2^x$



1-10 obvious

(11) $\frac{5+2}{4-1} = \frac{7}{3}$

$y+2 = \frac{7}{3}(x-1)$

$y = \frac{7}{3}x - \frac{13}{3}$

$13 = \frac{7}{3}x - \frac{13}{3}$

$39 = 7x - 13$

$52 = 7x$

$\frac{52}{7} = x$

NO. Doesn't satisfy same equations,

(12) $y = 2x+1$ $\quad 2x+1 = 5x-3$

$y = 5x-3$ $\quad 4 = 3x$

$\quad -3x+4$ $\quad \frac{4}{3}$

$x = \left(\frac{4}{3}, \frac{11}{3}\right)$ $\quad \frac{8}{3} + \frac{3}{3}$

(13) a) $a_1 \neq a_2$

$y = a_1x + b_1$

$y = a_2x + b_2$

$0 = (a_1 - a_2)x + b_1 - b_2$

$x = \frac{b_2 - b_1}{a_1 - a_2}$ Not //

a) because this is the x coord where they cross.

13) If $a_1 = a_2$ then $b_2 = b_1$ as same line & same line is considered //.

(14) $y = ax + b$ $(1,4)$ $(2,-3)$

$4 = a + b$

$-3 = 2a + b$

$-8 = -2a - 2b$

$-11 = -b$

$b = 11$

$a = -7$

(15) a) obvious

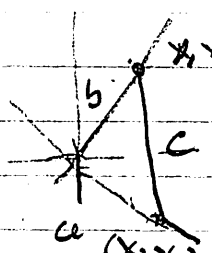
b) $(1, a_1)$ $(1, a_2)$

If $x_1x_2 + y_1y_2 = 0$ then $\perp$ so $1 + a_1a_2 = 0$ if $a_1a_2 = -1$

l_1 & l_2 $\perp$ if $a_1a_2 = -1$,

(16)

$y = \frac{-y_1}{x_1}x$ $\quad y = -\frac{y_2}{x_2}$



$a^2 + b^2 = c^2$ etc.

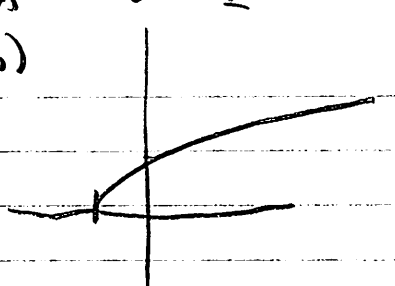
$x_2^2 + y_2^2 + x_1^2 + y_1^2 = x_1^2 - 2x_1x_2 + x_2^2 + x_1^2 - 2y_1y_2 + y_2^2$

$0 = x_1x_2 + y_1y_2$

Guided Quiz on chart 2

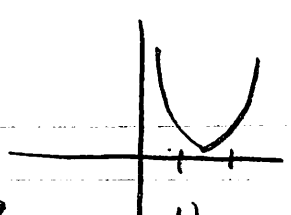
① a) $x \in \mathbb{R}$ $x \geq -1$

b)



② a is coefficient of x in the form of $mx+b$

③ a) $-\frac{1}{3}$ b) -1, c) 0



④ $f(x) = 3x^2 - 9x + 7$

$\frac{12}{7}$ $\frac{7}{84}$

$\frac{9 \pm \sqrt{81 - 84}}{6}$

$\frac{9}{6} = \frac{3}{2}$ $7 - \frac{81}{12}$

$7 - \frac{27}{4} = \frac{1}{4}$

No.

⑤

a) $\frac{12-3}{1} = 9$

b) $\frac{1.331 - 1}{.1} = 3.31$

⑥

a) $\frac{(1+h)^2 - 1}{h} = \frac{1+2h+h^2-1}{h} = 2+h$

B $\frac{8(1+h) - 3 - 5}{h} = \frac{8+8h-8}{h} = 8$

C) $\frac{\frac{5}{1+h} - 5}{h} = \frac{5 - 5 - 5h}{h(1+h)} = \frac{-5}{(1+h)}$

D) $\frac{(1+h)^3 + 2(1+h) - 3}{h} = \frac{1 + 3h + 3h^2 + h^3 + 2 + 2h - 3}{h}$

$h^2 + 3h + 5$

$(1+h)((1+h)^2 + 2) - 3$

$(3 + 2h + h^2) - 3$

$1 + h$

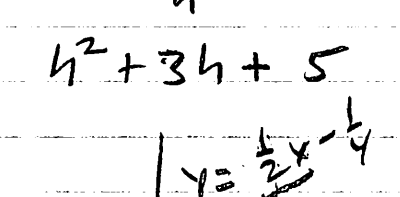
$3 + 2h + h^2$

$-3 \quad 3h + 2h^2 + h^3$

$5h + 3h^2 + h^3$

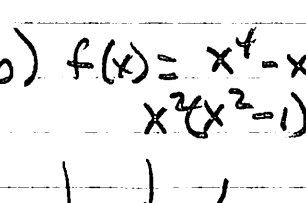
⑦

a)

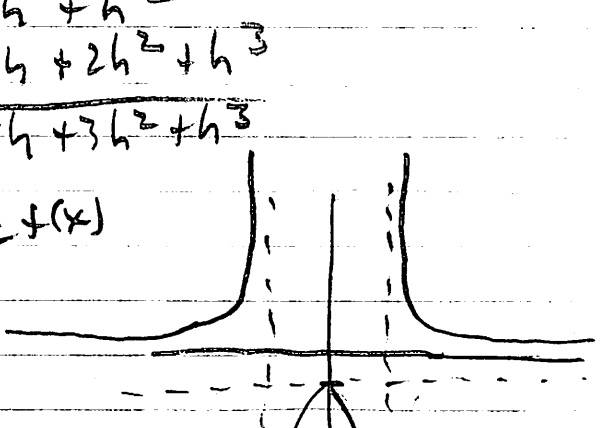


b) $f(x) = x^4 - x^2$

$x^2(x^2 - 1)$



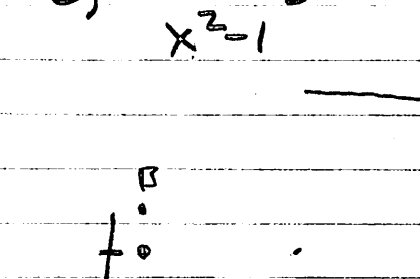
c) $\frac{1}{x^2 - 1} = f(x)$



⑧ a) Sure

b) No Not fctn

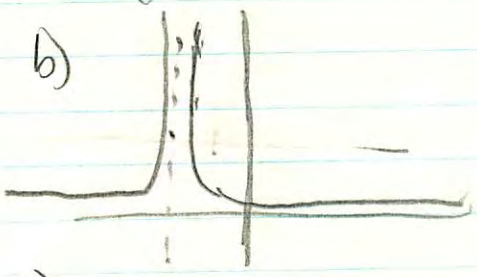
c) Yes $m=0$



Review

① a $\frac{1}{(x+2)^2} = f(x)$

b)



c) $x \in \mathbb{R}\ x \neq -2$

② a) $\frac{x_2^2+5-x_1^2-5}{x_2-x_1} = x_2+x_1$

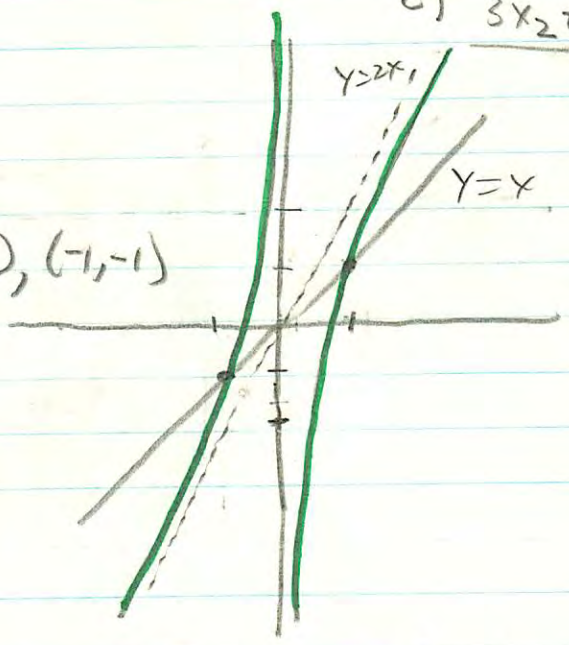
b) $\frac{\frac{-1}{x_2^2}+\frac{1}{x_1^2}}{x_2-x_1} = \frac{\frac{-x_1^2+x_2^2}{x_1^2x_2^2}}{x_2-x_1} = \frac{+(x_2+x_1)}{x_1^2x_2^2}$

c) $\frac{3x_2+2-3x_1-2}{x_2-x_1} = 3$

③ $2x-\frac{1}{x}$

$f(x) = 2x - \frac{1}{x}$

Intersects $(1,1), (-1,-1)$



④ a) 1 b) 1 c) −1

⑤ $f(a+b) = (a+b)^2 = a^2+2ab+b^2 = f(a)+f(b)+2ab.$

⑥ a) $-3a+2 \neq 3a+2$ NO

b) $(-a)^2 = a^2$ YES

c) $(-a)^3+(-a) \neq a^3+a$ NO

D) $(-a)^2+(-a) \neq a^2+a$ NO

E) $\frac{1}{-a} \neq \frac{1}{a}$ NO

f) $(-a)^4-5|-a| = a^4-5a$

⑦ $y = -\frac{1}{2}x + \frac{3}{2}$

⑧ $\frac{5+2}{2} = \frac{7}{2}$

$y = \frac{7}{2}x - 2$

(9) a) $|a|$ very large.
b) $|a|$ very small
c) $a < 0$
d) $a > 0$

(10) a) $2.75 - 0 = 2.75$
b) $.7143 - \frac{2}{3} = .0476$

(11) $f(f(2))$ $\quad f(-x) = x$
a) $f(\frac{1}{2}) = 2$ B 2
c) $f(x) = x^9 = 512$ D 10

(12) a) (0, 2)
b) (1, 0)
c) (2, -2)
d) (3, 0)

(13)
a) 2
b) 4
c) 5

The world famous Canadian big league power hitter, Jasper leDog stepped up to the plate. He popped up the ball. He made contact with it 3 feet above the ground. The ball soared almost out of sight!

- What is the height of the ball after 2 seconds?
- When is the ball 80 feet above the ground?
- If the catcher misses the ball, when does it hit the ground.
- If the six-foot tall catcher catches the ball one foot above his head, when does it land in his mitt?
- How high is the ball at its highest point? When did it get there?

We will use the following function of height in terms of time: $h(t) = -16t^2 + 98t + 3$. The $-16t^2$ represents the effect of gravity on the ball. The $+98t$ represents the vertical velocity of the ball. That is, how hard the ball was hit straight up. Finally the $+3$ represents that the ball was hit by the bat 3 feet above the ground.

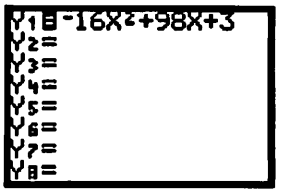
Turn on your calculator by pressing the ON button in the lower left hand corner.

We need to key in our function so press Y= in the upper left corner. The cursor is blinking on the Y_1 line. On your calculator there is a "subtract" key and a different key called "minus." The "subtract" key is in the right hand column between the "times" and "add" keys. The "minus" key is on the bottom row.

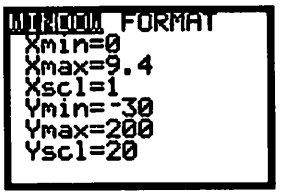
Press "minus"
Press 16
Press x, t, θ (third row, second column)

Press x^2 (row 6, column 1)
– – – – – – OR – – – – – –
Press ^ (above the + key)
Press 2

Press "plus" (above the ENTER key)
Press 98
Press x, t, θ
Press "plus"
Press 3
Your screen should look like this:



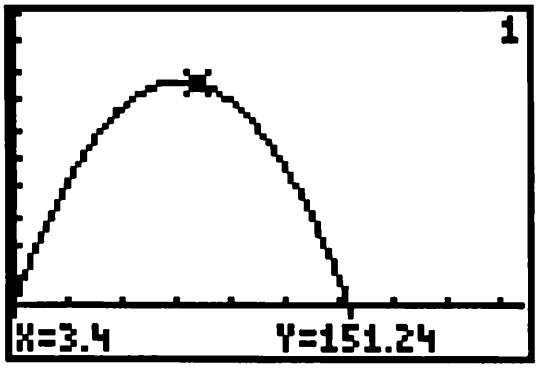
Press WINDOW
Press Down Arrow (or ENTER)
Press zero
Press ENTER
Press 9.4 etc. to get :



The screen width is 94 pixels so we like some decimal multiple of 94. The screen height is 62 pixels but we will not need to make adjustments for that at this time.

The first question was about the height of the ball after 2 seconds. We have 4 different methods to find out.

Method 1. Graphically: Press TRACE



Using the left arrow, move the cursor left until the x–value is 2. The y–value is the height desired.

Method 2. Using function notation in the HOME SCREEN.

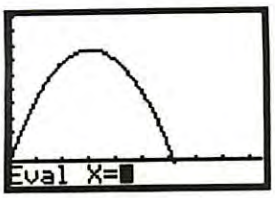
Press 2nd QUIT (next to the blue 2nd button) This takes you to the Home Screen.
Press 2nd Y-VARS This lists the different kinds of functions available. The first item is the standard function and is highlighted so press ENTER.
This lists all the Y functions in the calculator. Since we only have one function, and it is already highlighted ...
Press ENTER This places the function Y_1 on the Home Screen.
Press (6th row, 3rd column
Press 2) ENTER

The function value is 135.

Method 3. Calculate the value using the CALC menu.

Press 2nd CALC (1st row 4th column)

Press ENTER (This selects option 1 which is value.



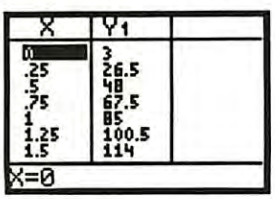
Press 2 ENTER

Method 4. Use a table of values.

Press 2nd TblSet row 1 column 2

Use the ENTER and number and arrow keys to make your screen look like this:

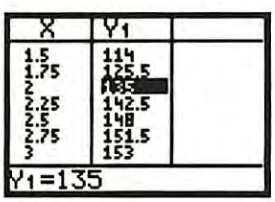
Press 2nd TABLE Row 1 Column 5



X	Y1
0	3
.25	26.5
.5	48
.75	67.5
1	85
1.25	100.5
1.5	114

X=0

Press the arrow keys until you see the desired values.



X	Y1
1.5	114
1.75	125.5
2	135
2.25	142.5
2.5	148
2.75	151.5
3	153

Y1=135

The next question was when was the ball 80 above the ground. There 2 ways to compute this. First we will define a second function, $f(t) = 80$.

Press Y=

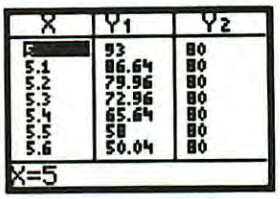
Press ENTER or use the down arrow to get to the second function.

Press 80 Notice that the equal sign "inverted" when you put in the 80. When the equal sign is inverted, the function is active. To turn off or turn on a function, place the cursor on the = sign using the arrow keys and then press ENTER.

Press 2nd QUIT to get out of the function definition and to return to the Home Screen.

ethod 1. We will zoom in using our table options.

Press and hold the down arrow key. Watch the values of Y_1 and Y_2. When are they the same or very nearly the same?



X	Y1	Y2
5	93	80
5.1	86.64	80
5.2	79.96	80
5.3	72.96	80
5.4	65.64	80
5.5	58	80
5.6	50.04	80

X=5

5.2?

Press 2nd TblSet and make ΔTbl = .01

Press 2nd TABLE.

Use the arrow keys again to get Y_1 and Y_2 about equal.

Press 2nd TblSet and make ΔTbl = .001

Press 2nd TABLE.

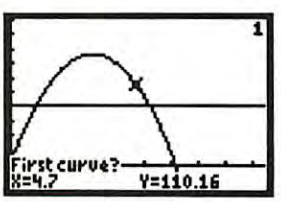
Use the arrow keys again to get Y_1 and Y_2 about equal.

Continue until you have an answer accurate to 4 digits.

Method 2. Using the Calculate Menu:

Press 2nd CALC

Move the cursor down to "intersect" and press ENTER or just press 5.



Press ENTER two times.

These identify the two functions of interest since it is possible for us to have many more than 2 functions on our screen.

Move the cursor near the intersection point you want the calculator locate. Press ENTER.

The third question concerned when the ball would hit the ground. Three methods are available.

Method 1. Zoom in using the table as we did in method 1 of question 2 looking for the Y_1 value to be zero.

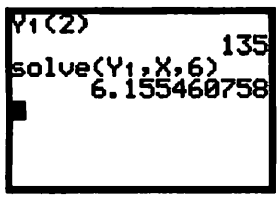
Method 2 Calculate the root of Y_1 since we are interested when the function is zero.

Press 2nd CALC. Press 2.

Press Enter to select that x value as lower bound.
Thenu se the right arrow to move the cursor so that it is *below* the x–axis. Press ENTER.
Move the cursor to the x–axis to select it as your "guess" and press ENTER.
(x = 6.1554608)

Method 3. We will use a programming instruction to do the job.

Press 2nd QUIT to get to the Home Screen.
Press MATH row 4 column 1
Press the down arrow to see all the options. Select the option solve(by highlighting it and then pressing ENTER.
Locate your supplement commands for your calculator and find solve(.
Solve(*expression,var,guess*)
NOTE the comma is above the 7 button, the variable X comes from the 3rd row 2nd column and be sure to close your parenthesis and then press ENTER. We want Y_1. Start by pressing 2nd Y–VARS. Finish up your line. :



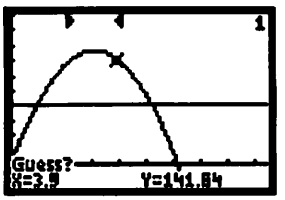
The fourth question about the catcher who does catch the ball you can do.

The fifth question was how high did the ball get and when did it do it?

Method 1 Use TblSet and TABLE and the method of zooming in on the largest value of Y_1.

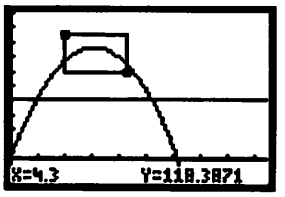
Before we begin this next method, I would like to "save" the current view so I can get back to it easily.
Press ZOOM. Press right arrow to highlight MEMORY.
Press 2. This stores the current window configuration for our later recall using option 3 from the list of choices ZoomRcl.

Method 2 Use the CALC function maximum.
After first moving the cursor on the left of the apparent maximum value location, pressing ENTER and moving the cursor to the right of the max location and pressing ENTER again:



Move the cursor to the apparent max and press ENTER a third time.

Method 3 We will look at the graph of the function and zoom in so we can see the exact spot.
Press TRACE
Move the cursor so it is on the obvious maximum spot. Since we are not close enough, it is hard to tell what that height is.
Press ZOOM. Notice that the first line is highlighted. You can choose an option by pressing ENTER when the option is highlighted or by just pressing the number on the left of the option you want.
We want ZBOX. (Zoom Box). The cursor turns into a + sign. Move the + sign left of the high point a little and then up a little and press ENTER.
Move the cursor to the right and below of the high point:



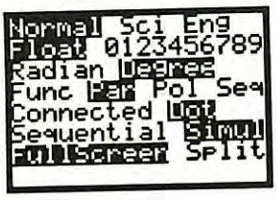
Press ENTER the second time and the calculator will make the necessary WINDOW adjustments so that the box you made will fill the viewing screen.
Repeat the ZBox routine a couple of times. You should get close enough that the "curve" will look like a horizontal line. You should be at that maximum value.

Just for fun ...
Press WINDOW. look at the values.
Press ZOOM, select MEMORY and Press 3. We have restored our window.
Press WINDOW. look at the values now.
Press MODE, select SPLIT and then press GRAPH.
To see different possibilities for the bottom of the screen, press the following keys one at a time:

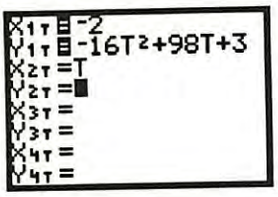
Motion Problems Using Parametric Equations

The graphing calculator can represent motion as parametric equations and show the position of an object in its flight path. Motion is assumed to be in a plane and air resistance is ignored.

Press MODE:



Press Y=:

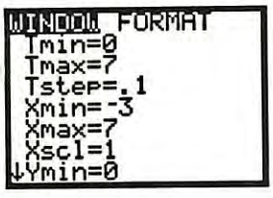


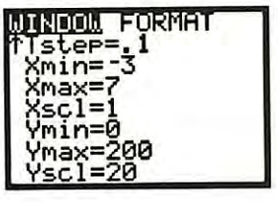
We want Y_{1T} where the cursor is now sitting.

Press 2nd Y-VARS, then 2 to select parametric and then 2 to select Y_{1T}.

The Y_{1T} is placed properly and the pair of equations becomes active (= signs are inverted).

Press WINDOW:





Now you see both the path of the ball (on the right) but also the height vs. time graph (on the left).

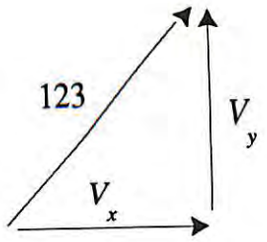
The "Green Monster," the left-field wall in Fenway Park in Boston, is 37 feet high and extends 210 feet towards center field, where its distance from home plate is 379. A baseball is hit by Jake leSnake (another famous canadian baseball player) when it is 3 feet from the ground with an initial velocity of 123 feet per second at an angle of elevation of 32 degrees.

A wind of 3 mph (4.4 ft/sec) is blowing towards the batter. The ball is on a path that will take to the deepest part of the Green Monster, 379 feet away. Is it a home run? What is the smallest angle that would produce a home run? What angle would produce a ball which could be caught?

initial force is resolved into the horizontal (x) and vertical (y) components using trigonometry:

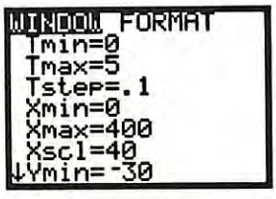
$$V_x = 123\cos 32^\circ \text{ and } V_y = 123\sin 32^\circ$$

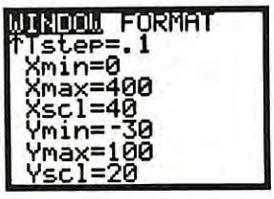
where:



The line X_{1T} means the wall is 379 from home plate.

You make the ≤ symbol using 2nd TEST (row 4) The wall is 37 feet tall so we want the calculator to plot points only up to 37 feet high. When T is larger than 1, 37T would be larger than 37 and the expression (T≤1) would be false. This is interrupted by the calculator as "equal to zero." That zero times the 37T still equals zero so the coordinate (379,0) gets turned on over and over. Until T = 1, (T≤1) is true and this is interpreted as a factor equal to plus one.





Press GRAPH to see the path of the ball.

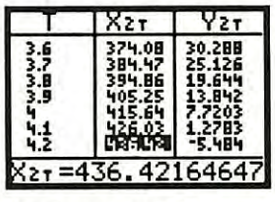
Was it a home run?

Next using TblSet set TblMin = 3 and ΔTbl = .1.

Press TABLE.

Press the right arrow 3 times to scroll the values of X_{2T} into view.

Scroll down to get:



T	X2T	Y2T
3.6	374.08	30.288
3.7	384.47	25.126
3.8	394.86	19.644
3.9	405.25	13.842
4	415.64	7.7203
4.1	426.03	1.2783
4.2	[illegible]	-5.484

X2T=436.42164647

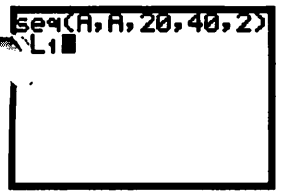
~ote that 3.6 is the number in the upper left corner. This number will be moved into the TabMin spot. Press 2nd TblSet and make ΔTbl = .01. Press TABLE.

Repeat the process to close in on the X values near 379.

There is another operation the calculator handles which is unusal and it has to do with lists of values.

Lets start on the Home Screen. Press 2nd Quit.

Press 2nd List row 3 column 3 Press 5 to get Seq(



To get the A, press ALPHA. Press A row 4 col 1.

To get the comma, press , above the 7.

The short arrow means "store" and the STO key is in row 9.

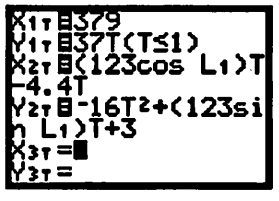
The L_1 means list # one and you get it by pressing 2nd 1.

When your screen looks like mine, press ENTER.

The meaning of the line is that we are first placing the value 20 in the variable A and placing that value in L_1 spot #1. Next we are adding 2 to A placing that value in L_1 spot #2 and so on until we get to 40.

To see the list itself and edit it, press STAT row 3 col 3 {NOTE you do not press 2nd STAT. I agree it seems logical to display a list by pressing LIST but the calculator eats lists as statistical data.} then press ENTER. You could edit any of the values in you list if you wanted to.

Next we need to edit our functions. Press Y=.



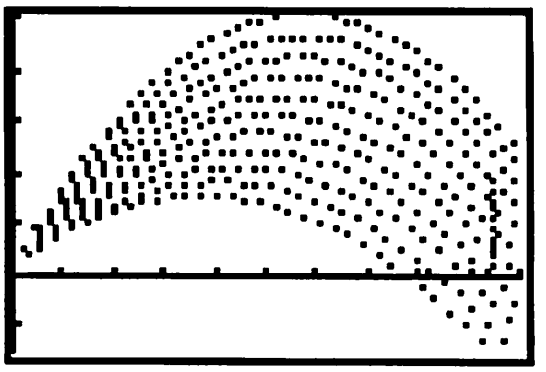
Notice the change in line X_{2T}.

I moved the cursor on top of the 3 in the 32 and pressed DEL row 2 col 3 two times, then I pressed 2nd INS row 2 col 3 also to put the calculator into insert mode. Next I pressed 2nd L_1.

Do the same for the next line.

What we have done is told the calculator to substitute the values from List #1 into our functions.

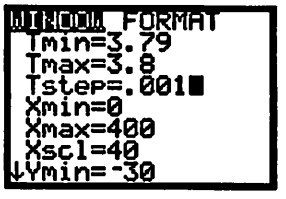
Press GRAPH.



Find the speed with which the ball in the original problem hits the wall and the angle it makes with the wall.

Change the L_1 back to 32 in X_{2T} and Y_{2T}.

Change WINDOW to T = [3.79, 3.8, .001] to get:



Use CALC *dy/dt*, *dx/dt*, and *dy/dx*.

Select *dy/dt* from CALC, trace to T = 3.794, the time that the ball hits the wall, and press ENTER to calculate.

Section 3.1
Pg 29-34

Read aloud the example problems & solutions.

Do Pblm 4 Pg 33 HW Pg 33-34 5, 8, 9, 11, 15-19

④ $[x, x_1]$ mass: $x_1^2 - x^2$ $\qquad \frac{x_1^2 - x^2}{x_1 - x} = x_1 + x$

$x_1 + x$ is density over that interval so at x we have $x + x = 2x$ if $\lim_{x_1 \to x} (x_1 + x) = 2x$

⑤ Sets up exactly the same as 4 above.

⑧ a) $[1, 1.1]$ $\qquad \frac{(1.1)^2 - 1^2}{1.1 - 1} = \frac{.21}{.1} = 2.1$

b) $[1, 1.01]$ $\qquad \frac{(1.01)^2 - 1^2}{1.01 - 1} = \frac{.0201}{.01} = 2.01$

c) $[1, 1.001]$ $\qquad \frac{(1.001)^2 - 1}{.001} = \frac{.002001}{.001} = 2.001$

d) the magnification appears to be approaching 2 ∴ it must be 2.

⑨ $[1, \tau]$ $\quad 2\tau^2 + \tau$ average speed is total dist ÷ by total time

a) at time 1 position is $2 + 1 = 3$ } total dist traveled is
at time τ " is $2\tau^2 + \tau$ } $2\tau^2 + \tau - 3$

Time of travel is $\tau - 1$

∴ average speed is $\frac{2\tau^2 + \tau - 3}{\tau - 1} = \frac{(2\tau + 3)(\tau - 1)}{\tau - 1} = 2\tau + 3$

b) $\lim_{\tau \to 1} 2\tau + 3 = 5$ ft/sec.

11) at x_1 $\quad y = 3x_1^2 - x_1 + 2$ $\qquad \frac{3x_1^2 - x_1 + 2 - 3x^2 + x - 2}{x_1 - x}$

a) $\frac{3x_1^2 - 3x^2 - (x_1 - x)}{x_1 - x} = \frac{3(x_1^2 - x^2) - (x_1 - x)}{x_1 - x} = \frac{3(x_1 - x)(x_1 + x) - (x_1 - x)}{x_1 - x}$

$\frac{(x_1 - x)\,(3(x_1 + x) - 1)}{x_1 - x} = 3x_1 + 3x - 1 \Big]_{x_1 = x} \Rightarrow 6x - 1$

b) $\left(\frac{1}{6}, \frac{23}{12}\right)$

(15) $\left.\dfrac{\frac{1}{2}^3 - x^3}{\frac{1}{2} - x}\right\}_{x=\frac{1}{2}} = \left.\dfrac{\left(\frac{1}{2} - x\right)\left(\frac{1}{4} + \frac{1}{2}x + x^2\right)}{\frac{1}{2} - x}\right\}_{x=\frac{1}{2}} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$

the magnification is $\frac{3}{4}$.

(16) $\dfrac{(2.01)^2 - 2^2}{2.01 - 2} = \dfrac{.0401}{.01} = 4.01$ a) The bacteria grows .0401 in that length of time.

b) rate of growth is 4.01

c) rate of growth at $t = 2$ is 4.

(17) a) $\dfrac{(3.1)^2 - 3^2}{3.1 - 3} = \dfrac{.61}{.1} = 6.1$

b) $\dfrac{(3.01)^2 - 3^2}{.01} = \dfrac{.0601}{.01} = 6.01$

c) 6.

(18) $\dfrac{x^2 - 1^2}{x - 1} = x + 1 = m$ of tangent line, at $x = 1$ $m = 2$

$m = 2$, $(1, 1) \Rightarrow$ $y - 1 = 2(x - 1) \Rightarrow y = 2x - 1$

Question does $6, 12$ satisfy $y = 2x - 1$ No,

(19) we need the tangent line to satisfy the pt $(4, 9)$

required pt is (x_1, x_1^2). $\dfrac{x_1^2 - x^2}{x_1 - x} = x_1 + x = m$ of tangent line.

at x_1 is $2x_1$

$y - x_1^2 = 2x_1(x - x_1)$

$y - x_1^2 = 2xx_1 - 2x_1^2$

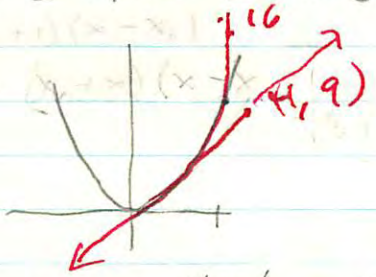
$y = 2xx_1 - x_1^2$

$x = 4$, $y = 9$ $\rightarrow$ $9 = 8x_1 - x_1^2$

$x_1^2 - 8x_1 + 9 = 0$

$x_1 = \dfrac{8 \pm \sqrt{64 - 36}}{2}$

$x_1 = \dfrac{8 \pm 2\sqrt{7}}{2} = 4 \pm \sqrt{7}$



Since we want to have the curve $\frac{1}{2}$ cross 4, 9 we must exit before $x = 4$ ie at $x = 4 - \sqrt{7}$. This coordinates are $(4 - \sqrt{7}, 23 - 8\sqrt{7})$.

19b. $(4, -9)$

(fm 19a) $Y = 2XX_1 - X_1^2$

$-9 = 8X_1 - X_1^2$

$X_1^2 - 8X_1 - 9 = 0$

$(X_1 - 9)(X_1 + 1) = 0$

$X_1 = +9 \quad X_1 = -1$ Coord of required Pt is $(-1, 1)$.

Reject.

3.2 Pg 37-38

Notice that the proof is based on the identity

$$c^N - d^N = \left(c^{N-1} + c^{N-2}d + c^{N-3}d^2 + \ldots + d^{N-1}\right)(c-d)$$

this identity is only valid for $N > 0$, $N \in \mathbb{Z}$!

HW pg 40 3, 7, -11, 13, 15-20

1. Routine

3. write: $\left.\frac{d(5x^3 - x^2 + 2x + 5)}{dx}\right]_{x=1}$ etc.

6

8 routine.

9. using Statements pg 36 & pg 38 we need

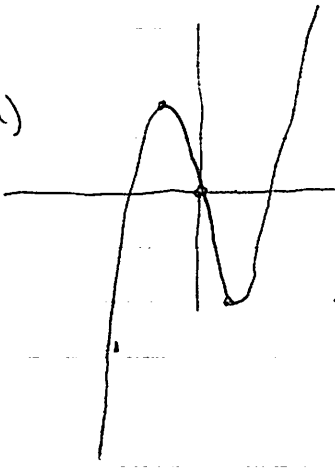
$\left.\frac{d(2x^5 + x)}{dx}\right]_{x=3} \Rightarrow \left. 10x^4 + 1\right]_{x=3} = 811$

11.

13. $y = x^5 - 5x$

x	y	
-1	4	
0	0	
1	4	
-2	-22	as $x \to -\infty$ $y \to -\infty$
2	22	as $x \to \infty$ $y \to +\infty$

a)



Notice that we have not proven Location of relative max or min!

b) $x^5 - 5x \overset{\text{set}}{=} 0$

$x(x^4 - 5) = 0$

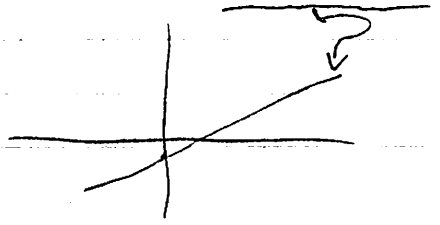
$x = 0 \quad x^2 = \pm\sqrt{5} \Rightarrow \quad x^2 = \sqrt{5} \quad x = \pm\sqrt[4]{5}$

c) $y' = 5x^4 - 5 \overset{\text{set}}{=} 0$

$x = \pm 1 \quad \therefore \quad (1, -4) \quad (-1, 4)$

15. $y' = 4x^3\Big]_{x=\frac{1}{2}} = \frac{1}{2} \qquad y - \frac{1}{16} = \frac{1}{2}\left(x - \frac{1}{2}\right) \qquad y = \frac{1}{2}x - \frac{3}{16}$

16. $\left.\frac{x^5 - 1}{x^3 - 1} = \frac{(x-1)(x^4 + x^3 + x^2 + x + 1)}{(x-1)(x^2 + x + 1)}\right]_{x=1} = \frac{5}{3}$



17. must simplify and

20

Hw 1 – 12, all routine.

14. a) $\dfrac{f(x_1)-f(x)}{x_1-x} = \dfrac{\frac{1}{x_1^3}-\frac{1}{x^3}}{x_1-x} = \dfrac{x^3-x_1^3}{(x_1-x)(x_1^3x^3)} =$

$\dfrac{(x_1-x)(x_1^2+x_1x+x^2)}{(x_1-x)(x_1^3x^3)} = \dfrac{x_1^2+x_1x+x^2}{x_1^3x^3}$

b) $\lim_{x_1\to x} \dfrac{x_1^2+x_1x+x^2}{x_1^3x^3} = \dfrac{3x^2}{x^6} = \dfrac{3}{x^4}$

15. explain completely.

16. a) $\dfrac{\frac{1}{.6}-\frac{1}{.5}}{.6-.5} = \dfrac{\frac{.5-.6}{.3}}{.1} = \dfrac{-.1}{(.1)(.3)} = -3.33.$

b) $\left(\frac{1}{x}\right)' = -\frac{1}{x^2} = \frac{-1}{.5^2} = \frac{-1}{\frac{1}{4}} = -4$

c) $\dfrac{\frac{1}{2.1}-\frac{1}{2}}{.1} = \dfrac{\frac{2-2.1}{4.2}}{.1} = \dfrac{-.1}{(.1)(4.2)} = -\frac{1}{4.2} = -.238$

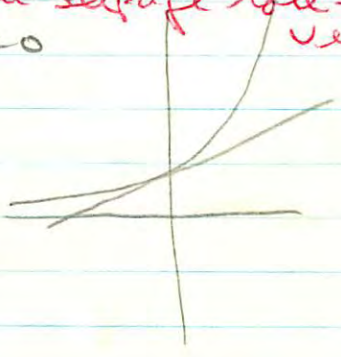
D) $-\dfrac{1}{x^2}\Big]_{x=2} = -.25$

17. a) Yes b) No. c) No

19. a) Since $\frac{1}{2\sqrt{t}}$ is $(\sqrt{t})'$ then it means the Rate the water is seeping into the ground rises.

b) when t is large the rate is very slow. This is why a lake is formed, eventually. The seepage rate is very small.

20. E) 'g' below evaluated at $x=0$



F)

g) $\lim_{h\to 0} \dfrac{10^{x+h}+10^x}{h} = \lim_{h\to 0} \dfrac{10^x\cdot 10^h+10^x}{h} =$

$\lim_{h\to 0} \dfrac{10^x(10^h+1)}{h} = C10^x$

3.3
Pg 40 - 52

HW Pg 48 14 - 20

Read Summary

Pg 50 Guided Quiz. 1-7

Pg 50-51 1-4, 7, 9-13, 15-24 25-30 Calculator

Guide Quiz all routine.

① Generally, The derivative measures how fast the value of the fctn is changing.

⑥ a) The tangent line to a curve f at a point is the line through that point which has the same slope as the derivative of f at that point.

b) Velocity is a directed speed, it is the derivative of f when f shows the position of a point wrt time.

c) Speed Absolute value of velocity.

Review exercises for chapt 3
all exercises are straight forward

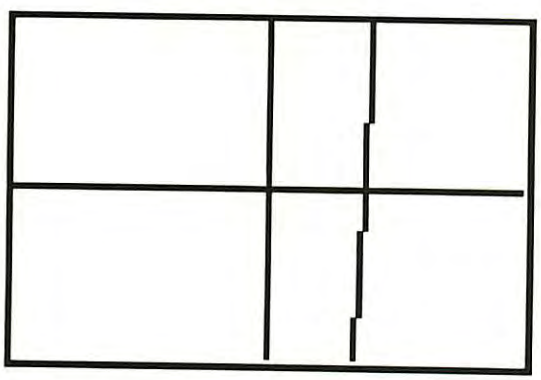
Window on the World

Mr. Mumaugh

Given $f(x) = x^3 - 2x^2 + x - 30$ 85 & 82 HAS 's' means

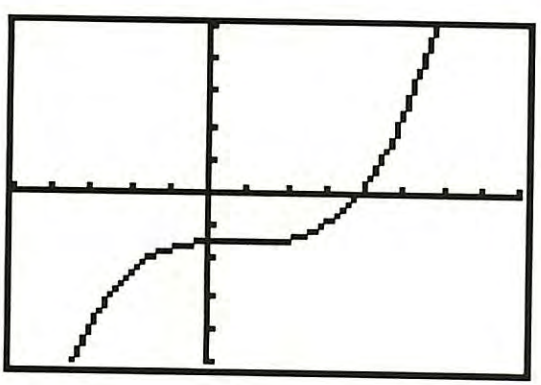
Determine the "window" settings which gives the supplied picture. Your picture must be exact including the "ticks" on the axes. (hint: YES, these are all of the *same* function as stated above!

a)



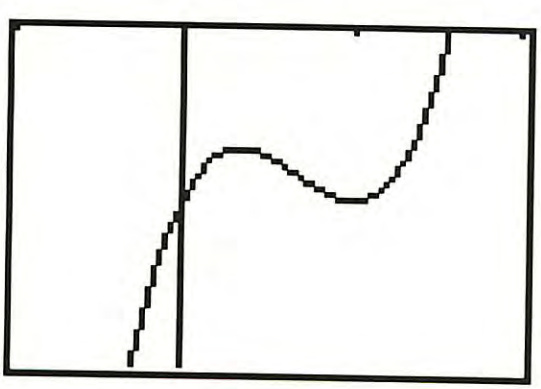
WINDOW FORMAT
Xmin= -10
Xmax= 10
Xscl= 0
Ymin= -10
Ymax= 10
Yscl= 0

b)



WINDOW FORMAT
Xmin= -5
Xmax= 8
Xscl= 1
Ymin= -100
Ymax= 100
Yscl= 20

c)



WINDOW FORMAT
Xmin= -1
Xmax= 2
Xscl= 1
Ymin= -30.5
Ymax= -29.5
Yscl= 30

II What are the coordinates of the axis intercepts of the function? (0, -30) (3.807126, 0)

III What are the coordinates of the relative max and the relative min shown in the graph of I c?
Min (1, -30) Max ($\frac{1}{3}$, -29.85185)

IV Why was this particular problem assigned? (What did you learn from it?) Be clear.
cannot tell the behavior of this fcto from only the graph. You need to know about derivative etc.

#27 p.58

Given $g_2 = \dfrac{g_1 + \frac{a}{g_1}}{2}$

given $g_1 > \sqrt{a}$ wTP ~~$g_2 > \sqrt{a}$~~

$g_1 > g_2$

start with given & build g_2

(1) $g_1 > \sqrt{a}$ Given

(2) $g_1 > \frac{a}{\sqrt{a}}$ def of $\sqrt{a}$

(3) ~~g_1~~ $\frac{a}{\sqrt{a}} > \frac{a}{g_1}$ $\div$ Larger #.

(4) $g_1 > \frac{a}{\sqrt{a}} > \frac{a}{g_1}$ combine Ins 2 & 3

(5) $2g_1 > g_1 + \frac{a}{\sqrt{a}} > g_1 + \frac{a}{g_1}$ add g_1

(6) $g_1 > \dfrac{g_1 + \frac{a}{\sqrt{a}}}{2} > \dfrac{g_1 + \frac{a}{g_1}}{2}$ $\div 2$

(7) $g_1 >$ $> g_2$ Sub given

use calculator only where absolutely necessary.

4.1
Pg 53 - 59

① a) $64^{1/2} = 8$ b) $64^{1/3} = 4$ c) $64^{-1/3} = \frac{1}{4}$ d) $64^{2/3} = 16$
e) $64^{-5/6} = \frac{1}{32}$

2

12

16 - 18

20 a) $x < x^2$ $|x| > 1$
b) $x^2 < x$ $0 < x < 1$
c) $x^2 < x^3$ $x > 1$
d) $x^3 < x^2$ $x < 1$ $x \neq 0$

21

22

a) $(a_1 \cdot a_2 \cdot a_3 \cdots a_k)(\quad) = I^2$

k PRIMES. k primes $\Rightarrow$ $2k$ primes

b)

N^2 is product of $2k$ Primes (ie an even # of Primes). multiply by the prime "2" to get $2N^2$. we have included one more prime $\therefore$ we have an odd # of Primes.

c) $\sqrt{2} = \frac{M}{N}$

$N\sqrt{2} = M$

$2N^2 = M^2$

odd # of Primes = even # of Primes $\therefore$ contradiction

Pg 59

This is reasonable since $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$

so each value of x is bigger than the preceding one.

Let's see if

$1 < 1 + \frac{1}{N+1}$

$\left(1+\frac{1}{x}\right)^N < \left(1+\frac{1}{N}\right)^N \left(1+\frac{1}{N+1}\right.$

(27) $g_2 = \dfrac{g_1 + \frac{a}{g_1}}{2}$

This is half of this

(A)

(B) $g_1 > \sqrt{a}$

1ST prove $g_1 > g_2$

$g_1 > \sqrt{a}$ given

$g_1 > \frac{a}{\sqrt{a}}$ def of $\sqrt{a}$

$g_1 > \frac{a}{\sqrt{a}} > \frac{a}{g_1}$ ÷ largest #

$2g_1 > g_1 + \frac{a}{\sqrt{a}} > g_1 + \frac{a}{g_1}$ add g_1

$g_1 > \frac{g_1 + \frac{a}{\sqrt{a}}}{2} > \frac{g_1 + \frac{a}{g_1}}{2}$ ÷ 2

$g_1 > \quad > \frac{g_1 + \frac{a}{g_1}}{2} = g_2$ Given

$g_1 > \sqrt{a} = \frac{\sqrt{a}}{\sqrt{a}} > \frac{\sqrt{a}}{g_1}$

$g_2 < \frac{g_1 + g_1}{2}$

Pg 59–65

approaches e from below

$\lim_{N\to\infty}\left(1+\frac{1}{N}\right)^N = e$

approaches e from abv.

$\lim_{N\to-\infty}\left(1+\frac{1}{N}\right)^N = e$

Skip / Go to lecture 3 pg 8.

$\lim_{N\to\infty}(1+x)^{\frac{1}{x}} = e$ where $x=\frac{1}{N}$

→ thm 1 $\left(1+\frac{1}{N}\right)^N < \left(1+\frac{1}{N+1}\right)^{N+1}$ $\forall N>0$ we will need a lemma before we can prove this thm.

lemma ① $c^{N+1}-d^{N+1} < (N+1)c^N(c-d)$ $c>d\geq 0$

② $c^{N+1}-d^{N+1} > (N+1)d^N(c-d)$

$$c^{N+1}-d^{N+1} = \underbrace{\left(c^N + c^{N-1}d + c^{N-2}d^2 + \cdots cd^{N-1} + d^N\right)}_{N+1 \text{ items}}(c-d)$$

replace d's with c in first factor. This makes the Right Side larger

$c^{N+1}-d^{N+1} \leq \left(c^N + c^N + c^N + \cdots c^N\right)(c-d)$

$c^{N+1}-d^{N+1} \leq (N+1)c^N(c-d)$

the second portion works the same way.

Thm 1 $\forall N>0$ $\left(1+\frac{1}{N}\right)^N < \left(1+\frac{1}{N+1}\right)^{N+1}$

let $c = 1+\frac{1}{N}$

$d = 1+\frac{1}{N+1}$

Clearly $c>d$

in ① abv $\left(1+\frac{1}{N}\right)^{N+1} - \left(1+\frac{1}{N+1}\right)^{N+1} < (N+1)\left(1+\frac{1}{N}\right)^N\left(1+\frac{1}{N}-1-\frac{1}{N+1}\right)$

do not eliminate $\left(1+\frac{1}{N}\right)^N$ since we need this for our thm

$\left(1+\frac{1}{N}\right)\left(1+\frac{1}{N}\right)^N - \left(1+\frac{1}{N+1}\right)^{N+1} < (N+1)\left(1+\frac{1}{N}\right)^N\left(\frac{N+1-N}{N(N+1)}\right)$

$< \left(1+\frac{1}{N}\right)^N\left(\frac{1}{N}\right)$

$\left(1+\frac{1}{N}\right)^N\left[\left(1+\frac{1}{N}\right)-1\right] < \left(1+\frac{1}{N+1}\right)^{N+1}\frac{1}{N}$

$\left(1+\frac{1}{N}\right)^N < \left(1+\frac{1}{N+1}\right)^{N+1}$

Thm 2 $\left(1+\frac{1}{N}\right)^N < 4 \quad \forall N > 0$

Let $c = 1+\frac{1}{2N}$ $d = 1$ agn $c > d$ Substitute into ① above.

$$\left(1+\frac{1}{2N}\right)^{N+1} - (1)^{N+1} < (N+1)\left(1+\frac{1}{2N}\right)^N \left(1+\frac{1}{2N}-1\right)$$

$$\left(1+\frac{1}{2N}\right)\left(1+\frac{1}{2N}\right)^N - 1 < (N+1)\left(1+\frac{1}{2N}\right)^N \left(\frac{1}{2N}\right)$$

$$\left(1+\frac{1}{2N}\right)\left(1+\frac{1}{2N}\right)^N - 1 < \left(\frac{1}{2}+\frac{1}{2N}\right)\left(1+\frac{1}{2N}\right)^N$$

$$\left(1+\frac{1}{2N}\right)^N \left(1+\frac{1}{2N}-\frac{1}{2}-\frac{1}{2N}\right) < 1$$

$$\left(1+\frac{1}{2N}\right)^N \left(\frac{1}{2}\right) < 1$$

$$\left(1+\frac{1}{2N}\right)^N < 2$$

$$\left(1+\frac{1}{2N}\right)^{2N} < 4$$

Squaring because we want the power to be the same as the denominator

Let $2N = x$

$\left(1+\frac{1}{x}\right)^x < 4$

Lecture opener * (Prior to HW on pg 64)

Given: $N > 0$ $D = 1 + \frac{1}{N+1}$ $C = 1 + \frac{1}{N}$ so $D < C$

1st I DO

question #1 $D^N < C^N$? True.

question #2 $D^{N+1} > C^N$? True ... ? maybe? False?

* Before this I'd first show answer to problem 22 pg 58

(sets up the "how many items" thing used in this proof)

then go

Lecture review of HW on pg 64

Given $C \geq D > 0$;

$? \leq C^{N+1} - D^{N+1} \leq ?$

IT factors thus ;

$(C-D)(C^N + C^{N-1}D + C^{N-2}D^2 + \ldots + CD^{N-1} + D^N)$

If we replace C's in 2nd factor w/ Ds we get a smaller quantity because we replaced an item with a smaller item.

$(C-D)(D^N + D^{N-1}D + D^{N-2}D^2 + \ldots + D^N)$

How many items $N+1$

$(C-D)(N+1)D^N \leq C^{N+1} - D^{N+1}$

Like wise for right side. Tend with

$\star \quad (C-D)(N+1)D^N \leq C^{N+1} - D^{N+1} \leq (C-D)(N+1)C^N$

Let $C = 1 + \frac{1}{N}$ and $D = 1 + \frac{1}{N+1}$

notice $C > D$ as given.

we substitute into right $\frac{2}{3}$ of $\star$

$$c^{n+1} - d^{n+1} \leq (c-d)(n+1)\, c^n$$

$$\left(1+\tfrac{1}{n}\right)^{n+1} - \left(1+\tfrac{1}{n+1}\right)^{n+1} \leq \left(1+\tfrac{1}{n} - \left(1+\tfrac{1}{n+1}\right)\right)(n+1)\left(1+\tfrac{1}{n}\right)^n$$

$$\leq \left(\tfrac{1}{n} - \tfrac{1}{n+1}\right)(n+1)\left(1+\tfrac{1}{n}\right)^n$$

$$\leq \left(\frac{n+1-n}{n(n+1)}\right)(\quad)(\quad)^n$$

$$\leq \left(\frac{1}{n(n+1)}\right)\cancel{(n+1)}\left(1+\tfrac{1}{n}\right)^n$$

$$\left(1+\tfrac{1}{n}\right)^{n+1} - \left(1+\tfrac{1}{n+1}\right)^{n+1} \leq \frac{1}{n}\left(1+\tfrac{1}{n}\right)^n$$

$$\left(1+\tfrac{1}{n}\right)^{n+1} - \frac{1}{n}\left(1+\tfrac{1}{n}\right)^n \leq \left(1+\tfrac{1}{n+1}\right)^{n+1}$$

$$\left(1+\tfrac{1}{n}\right)^n\left[\left(1+\tfrac{1}{n}\right) - \tfrac{1}{n}\right] \leq$$

$$\left(1+\tfrac{1}{n}\right)^n \leq \left(1+\tfrac{1}{n+1}\right)^{n+1}$$

C ≤ D

Thus

a larger # to a ≤ a smaller # to a

our text uses this to prove $e \leq 4$.
My only regret is that the authors wasted such a nice proof on such a problem.

continuing . . .

useful Def of e $\quad \lim_{*\to\infty} \left(1+\frac{1}{*}\right)^{*} = e$

$\lim_{x\to\infty} \left(1+\frac{1}{x}\right)^{x} = e$ $\quad (1 + \text{nothing})^{\text{same}}$

$\lim_{x\to-\infty} \left(1+\frac{1}{x}\right)^{x} = e$ will be proved as HW problem. You must make each Problem in the HW make sense!

$\lim_{x\to 0} (1+x)^{\frac{1}{x}}$

Pg 64 #8 $\quad \text{Amount} = \text{Principle}\left(1 + \frac{\text{Interest Rate}}{\#\text{ Times Per Year Compounded}}\right)^{(\text{Years})\#}$

a) $1\left(1+\frac{.05}{N}\right)^{1\cdot N}$

b) $\left(1+\frac{1}{20N}\right)^{N}$ ← Remember How we constructed e.

$\lim_{N\to\infty}\left\{\left(1+\frac{1}{20N}\right)^{20N}\right\}^{\frac{1}{20}}$ $\quad \#$ compounded per yr.

$e^{\frac{1}{20}} = e^{.05}$

END of Lecture

4.2
pg 64-65

① $(1+\frac{1}{N})^N$ $N=5$ $\quad (1+\frac{1}{5})^5 = (\frac{6}{5})^5 = 2.49$

$N=-5$ $\quad (1-\frac{1}{5})^{-5} = 3.05$

② $(1+x)^{\frac{1}{x}}$

a) $x=\frac{1}{2}$ $\quad (1+\frac{1}{2})^2 = 2.25$

b) $x=\frac{1}{3}$ $\quad (1+\frac{1}{3})^3 = 2.37$

c) $x=-\frac{1}{2}$ $\quad (1-\frac{1}{2})^{-2} = 4.00$

d) $x=-\frac{1}{3}$ $\quad (1-\frac{1}{3})^{-3} = 3.38$

③ $(1-\frac{1}{N})^N$ a) $N=2$ $\quad (1-\frac{1}{2})^2 = .25$

b) $N=3$ $\quad (1-\frac{1}{3})^3 = (\frac{2}{3})^3 = \frac{8}{27} = .30$

c) $N=4$ $\quad (1-\frac{1}{4})^4 = (\frac{3}{4})^4 = .32$

④ a) $e = \lim_{N\to\infty} (1+\frac{1}{N})^N$ b) 2.718

⑤ a) $\lim_{N\to\infty} (1+\frac{1}{N})^{100} = \lim_{N\to\infty}\left[(1+\frac{1}{N})^N\right]^{\frac{100}{N}} = \lim_{N\to\infty} e^{\frac{100}{N}} = 1$ since $\frac{100}{N}\to 0$

b) $\lim_{N\to\infty} (1+\frac{1}{2N})^N$ $\quad (1+\frac{1}{2N})^N = \left[(1+\frac{1}{2N})^{2N}\right]^{\frac{1}{2}}$

$\lim (1+\frac{1}{2N})^N = e^{\frac{1}{2}}$

⑥ Unity

⑧ $A = P(1+\frac{i}{Q})^{NQ}$ P = Principle compounded Q times a yr, i = interest, N = # of yrs.

a) $A = 1(1+\frac{.05}{N})^N = (1+\frac{.05}{N})^N$

b) show $\lim_{N\to\infty} (1+\frac{.05}{N})^N = e^{.05}$

$(1+\frac{5}{100N})^N$

$(1+\frac{1}{20N})^N \Rightarrow \left[(1+\frac{1}{20N})^{20N}\right]^{\frac{1}{20}} \Rightarrow e^{\frac{1}{20}} = e^{.05}$

$N = -P$ $\quad\therefore N = -Q - 1$

$q = P - 1$ $\quad -N - 1 = Q$

$P = Q + 1$

$$\lim_{q \to \infty} \left(1 + \frac{1}{q}\right)^{q+1}$$

$$\lim_{q \to \infty} \left(1 + \frac{1}{q}\right)^{q} \left(1 + \frac{1}{q}\right)$$

$$e \qquad 1 \qquad = e$$

when $Q \to \infty$ then $N \to -\infty$

$$e = \lim_{N \to -\infty} \left(1 + \frac{1}{P-1}\right)^{P} = e$$ Sub N for Q and $P-1$ for Q

$$e = \lim_{N \to -\infty} \left(\frac{P-1+1}{P-1}\right)^{P} = \lim_{N \to +\infty} \left(\frac{P}{P-1}\right)^{P}$$ but $P = -N$

$$e = \lim_{N \to -\infty} \left(\frac{P}{P-1}\right)^{-N} = \lim_{N \to -\infty} \left(\frac{P-1}{P}\right)^{N} =$$

$$e = \lim_{N \to -\infty} \left(1 - \frac{1}{P}\right)^{N}$$ but $N = -P$

$$e = \lim_{N \to -\infty} \left(1 + \frac{1}{N}\right)^{N}$$

$\int \quad \int_R \quad \uparrow \sqrt{a}$

Given $\lim_{N\to\infty}\left(1+\frac{1}{N}\right)^N = e$

(10) $N = -P$

w+s: $\left(1+\frac{1}{N}\right)^N = \left(\frac{P}{P-1}\right)^P$

$\left(1+\frac{1}{-P}\right)^{-P}$

$\left(\frac{1-P}{-P}\right)^{-P}$

$\left(\frac{P-1}{P}\right)^{-P}$

$\left(\frac{P}{P-1}\right)^{P} = \left(\frac{P}{P-1}\right)^{P}$

(11) let $q = P-1$ so $P = q+1$

w+s $\left(\frac{P}{P-1}\right)^P = \left(1+\frac{1}{q}\right)^q\left(1+\frac{1}{q}\right)$

$\left(\frac{q+1}{q}\right)^{q+1}$

$\left(\frac{q+1}{q}\right)^{q}\cdot\left(\frac{q+1}{q}\right)$

$\left(1+\frac{1}{q}\right)^{q}\cdot\left(1+\frac{1}{q}\right)$

(12) a) $\lim_{q\to\infty}\left(1+\frac{1}{q}\right)^q \to e$

b) $\lim_{q\to\infty}\left(1+\frac{1}{q}\right) \to 1$

c) $\lim_{q\to\infty}\left(1+\frac{1}{q}\right)^q\left(1+\frac{1}{q}\right) \Rightarrow e\cdot 1$

$= e$

(13) w+s

$\lim_{N\to\infty}\left(1+\frac{1}{N}\right)^N = e$

$q\to\infty$

when q gets larger, P gets larger and N is a negative number with absolute value = P.

$\lim_{Q\to\infty}\left(1+\frac{1}{Q}\right)^{q+1} = e$

from (11) $q = P-1$

$\left(1+\frac{1}{P-1}\right)^P$

$\left(\frac{P}{P-1}\right)^P$

$\left(\frac{P}{P-1}\right)^{-N}$

$\left(\frac{P-1}{P}\right)^{N}$

$\left(1-\frac{1}{P}\right)^{N}$

$\left(1-\frac{1}{N}\right)^{N}$

$\lim_{N\to\infty}\left(1+\frac{1}{N}\right)^N = e$

(13) on next page

(13)

Problems 10 & 11 show: $\left(1+\frac{1}{N}\right)^N = \left(1+\frac{1}{q}\right)^q \left(1+\frac{1}{q}\right)$

where $q = p-1$ and $-N = p$.

as $N \to -\infty$ $q \to +\infty$ This becomes $e \cdot 1$

and $\lim_{q\to\infty}\left(1+\frac{1}{q}\right)^q = e$ by def.

⑨ $\lim_{N\to\infty}\left(\frac{N+2}{N+1}\right)^N = \lim_{N\to\infty}\left(1+\frac{1}{N+1}\right)^N$

$\lim_{N\to\infty}\left(\left(1+\frac{1}{N+1}\right)^{N+1}\right)^{\frac{N}{N+1}} =$

$\lim_{N\to\infty} e^{\frac{N}{N+1}}$; $\lim_{N\to\infty}\frac{N}{N+1} =$

$\lim_{N\to\infty}\frac{1}{1+\frac{1}{N}} = 1$ ∴ $\lim_{N\to\infty} e^{\frac{N}{N+1}} = \lim_{N\to\infty} e^{1-\frac{1}{N}} = e$

$N+1 \overline{)\,N}$ R -1 ; $\frac{N}{N+1} = 1-\frac{1}{N}$

⑩ $\left(1+\frac{1}{N}\right)^N = \left(\frac{P}{P-1}\right)^P$

$\left(1-\frac{1}{P}\right)^{-P} =$

$\left(\frac{P-1}{P}\right)^{-P}$

$\left(\frac{P}{P-1}\right)^P =$

$N = -P$

$\left(1+\frac{1}{P-1}\right)^{P-1}\left(1+\frac{1}{P-1}\right)$

$P-1 \overline{)\,P}$ R 1

$e \cdot 1 = e$

⑪ $q = P-1$ $\quad -P = N$

$q+1 = P$ $\quad P = -N$

$\left(\frac{P}{P-1}\right)^P = \left(1+\frac{1}{q}\right)^q\left(1+\frac{1}{q}\right)$

$= \left(1+\frac{1}{q}\right)^{q+1}$

$= \left(1+\frac{1}{q}\right)^q$

$= \left(1+\frac{1}{P-1}\right)^P$

$= \left(\frac{P}{P-1}\right)^P$

⑫ a) when q gets larger $\left(1+\frac{1}{q}\right)^q$ approaches e

b) $\left(1+\frac{1}{q}\right)$ goes to 1

c) $\left(1+\frac{1}{q}\right)^q\left(1+\frac{1}{q}\right)$ goes to $e \cdot 1 = e$

⑬ when q gets larger, P gets larger and N is a neg # with absolute value = P. Since

$\lim_{q\to\infty}\left(1+\frac{1}{q}\right)^{q+1} = e = \lim_{N\to-\infty}\left(1+\frac{1}{P-1}\right)^P = \lim_{N\to-\infty}\left(\frac{P}{P-1}\right)^P =$

$\lim_{N\to-\infty}\left(\frac{P}{P-1}\right)^{-N} = \lim_{N\to-\infty}\left(\frac{P-1}{P}\right)^N = \lim_{N\to\infty}\left(1-\frac{1}{P}\right)^N = \lim_{N\to-\infty}\left(1+\frac{1}{N}\right)^N = e$

method 1

notice

WTS

(14) $\lim_{N\to\infty} \left(1-\frac{1}{N}\right)^N = \frac{1}{e}$

$$\left(1-\frac{1}{N}\right)^N = \left(\frac{N-1}{N}\right)^N = \frac{1}{\left(\frac{N}{N-1}\right)^N}$$

Let $N-1=P$

$N = P+1$

$$\frac{1}{\left(\frac{P+1}{P}\right)^{P+1}} = \frac{1}{\left(1+\frac{1}{P}\right)^{P+1}}$$

$$\left(1+\frac{1}{P}\right)^{P+1} = \left(1+\frac{1}{P}\right)^{P}\left(1+\frac{1}{P}\right)$$

$\lim_{P\to\infty}$ $\Rightarrow$ $e \cdot 1 = e$

$$\therefore \lim_{P\to\infty} \frac{1}{\left(1+\frac{1}{P}\right)^{P+1}} = \frac{1}{e}$$

method 2 ← Best choice.

WTS $\lim_{N\to\infty} \left(1+\frac{1}{N}\right)^N = \frac{1}{e}$

$\lim_{N\to\infty} \left(1+\left(\frac{-1}{N}\right)^N\right)$

$\lim_{N\to\infty} \left(1+\frac{1}{-N}\right)^N$

$\lim_{N\to\infty} \left[\left(1+\frac{1}{-N}\right)^{-N}\right]^{-1}$

$e^{-1} = \frac{1}{e}$

method 3

$\lim_{N\to\infty} \left(1+\frac{1}{N}\right)^{N+1}$

$\lim_{N\to\infty} \left(\left(1-\frac{1}{N}\right)\left(1+\frac{1}{N}\right)^{N^2}\right)$

limits

4.3
Pg 65 -> 72

Pg 67

$$f'(x) = \lim_{x_1 \to x} \frac{f(x_1) - f(x)}{x_1 - x}$$

Note x is a fixed pt and x_1 is very close. as a matter of fact there is another fctn g which has x_1 in the domain and $\frac{f(x_1) - f(x)}{x_1 - x}$ as its range. ie $g(x_1) = \frac{f(x_1) - f(x)}{x_1 - x}$

Thus the derivative of f can be defined as

$$\lim_{x_1 \to x} g(x_1)$$

eg. $f(x) = x^3$ $\quad g(x_1) = \frac{f(x_1) - f(x)}{x_1 - x}$

$$= \frac{x_1^3 - x^3}{x_1 - x}$$

$$\lim_{x_1 \to x} \frac{(x_1 - x)(x_1^2 + x x_1 + x^2)}{x_1 - x} =$$

$$\lim_{x_1 \to x} \left(x_1^2 + x_1 x + x^2\right) = 3x^2 = f'(x)$$

Right & left limits ↓

Right & left hand limits

Eg 1 $\lim_{x \to 0^+} \frac{x}{|x|} = 1$ because we are approaching from the right if x is positive

$\lim_{x \to 0^-} \frac{x}{|x|} = -1$

$\lim_{x \to 0} \frac{x}{|x|}$ does not exist because left & right limits are not =.

Eg 2 $\lim_{x \to 0^+} \frac{2^{\frac{1}{x}} + 3}{2^{\frac{1}{x}} + 4} = \lim_{a \to +\infty} \frac{2^a + 3}{2^a + 4} = 1$

$\lim_{x \to 0^-} \frac{2^{\frac{1}{x}} + 3}{2^{\frac{1}{x}} + 4} = \lim_{a \to 0^+} \frac{\frac{1}{2^{\frac{1}{x}}} + 3}{\frac{1}{2^{1/x}} + 4} =$

$\lim_{b \to \infty} \frac{\frac{1}{2^b} + 3}{\frac{1}{2^b} + 4} = \frac{3}{4}$

$\lim_{x \to 0} \frac{2^{\frac{1}{x}} + 3}{2^{\frac{1}{x}} + 4}$ again does not exist. Because of Left & Right hand limits

4.3

Pg 71->72

1. $\lim_{x\to 3} \frac{x^2-3}{x+2} = \frac{6}{5}$

2. $\lim_{x\to 3} \frac{x^2-9}{x-3} = \lim_{x\to 3} x+3 = 6$

3. $\lim_{x\to 0^+} \sqrt{x} = 0$

4. $\lim_{x\to 0^-} \sqrt{x}$ does not exist.

5. $\lim_{x\to 3} \frac{\frac{1}{x}-\frac{1}{2}}{x-2} = -\frac{1}{6}$

6. $\lim_{x\to 2} \frac{\frac{1}{x}-\frac{1}{2}}{x-2} = \lim_{x\to 2} \frac{\frac{2-x}{2x}}{x-2} = \lim_{x\to 2} -\frac{1}{2x} = -\frac{1}{4}$

7. $\lim_{h\to 1} \frac{(1+h)^3-1}{h} = 7$

8. $\lim_{h\to 0} \frac{(1+h)^3-1}{h} = \frac{h^3+3h^2+3h+1-1}{h} =$

$\lim_{h\to 0} 3h^2+3h+3 = 3$

9. $\lim_{x\to 2^-} \sqrt{2-x} = 0$

10. $\lim_{x\to 1} \frac{x-1}{|x-1|} = 1$

11. $\lim_{x\uparrow 1} \frac{x-1}{|x-1|} = -1$

12. $\lim_{x\to 1} \frac{x-1}{|x-1|}$ doesn't exist.

13. $\lim_{x\to -2} \frac{x^3+8}{x+2} = \lim_{x\to -2} x^2-2x+4 = 12$

thus we can factor the top and then cancel out the $g(x)$ method. Notice $\lim_{x\to -2} \frac{x^3+8}{x+2}$ can be rewritten $\lim_{x\to -2} \frac{x^3-(-2)^3}{x-(-2)}$ So the function x^3

$c'(x) = 3x^2 \Big|_{x=-2} = 12$

14. $\lim_{x\to 4^+} \sqrt{x-4}+2 = 2$

15. $\lim_{x\to 0} \frac{\sqrt{x+4}-2}{x} \cdot \frac{\sqrt{\ }+2}{\sqrt{\ }+2} = \lim_{x\to 0} \frac{x+4-4}{x(\sqrt{x+4}+2)} =$

$\lim_{x\to 0} \frac{1}{\sqrt{x+4}+2} = \frac{1}{4}$

16. $\lim_{x\to a} f(x)$ exists $\forall a$ except $a \in \{1, 2\}$
$\lim_{x\to a^+}$ exists & $\lim_{x\to a^-}$ exists.

17. $\lim_{x\to a} f(x)$ exists everywhere.

18. $\lim_{x\to a} f(x)$ exists $\forall a$ except $a \in J$.
$\lim_{x\to a^+} f(x)$ & $\lim_{x\to a^-}$ exists

19. $\lim_{x\to a} f(x)$ exists $\forall a$

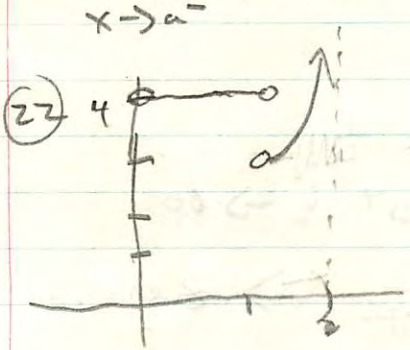
20. $\lim_{x\to a} f(x)$ exists $\forall a$

21. $\lim_{x\to a} f(x)$ exists $\forall a$ except $a=1$

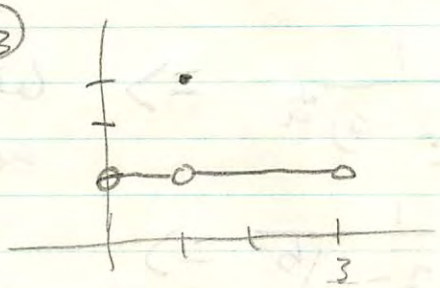
$\lim_{x\to a^+} f(x)$ exist $\forall a$

$\lim_{x\to a^-} f(x)$ exists $\forall a$ except $a=1$

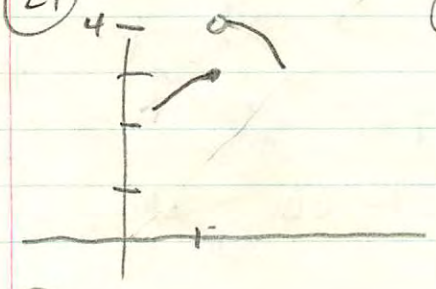
(22)



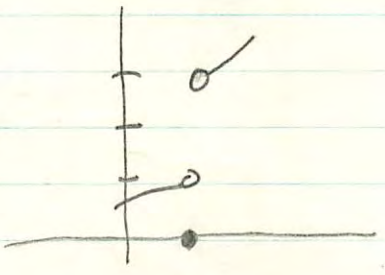
(23)



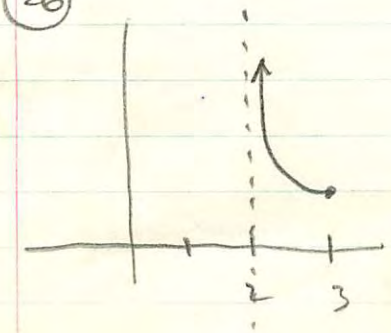
(24)



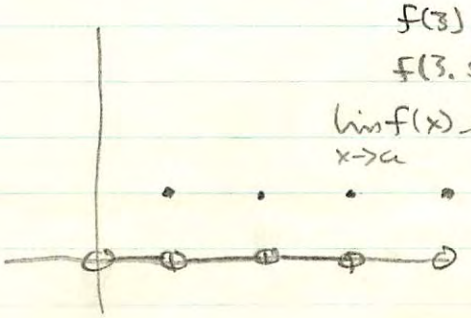
(25)



(26)



(27)



f(3) exists
f(3.5) exists
$\lim_{x\to a} f(x)$ exists $\forall a$

(11) $\lim_{x\to 0^+} (2+x)^{\frac{1}{x}}$

$\lim_{x\to 0^+} \left[(1+(x+1))^{\frac{1}{x+1}}\right]^{\frac{x+1}{x}}$

$\left[(1+1)^{1}\right]^{\frac{1}{0}}$ ← can't get rid of denominator. ∴ NO Limit

(12) $\lim_{x\to 0^-} (2+x)^{\frac{1}{x}}$ Let $a = -x$

as $x \to 0^-$ $a \to 0^+$

$\lim_{a\to 0^+} (2-a)^{\frac{1}{-a}}$

$\lim_{a\to 0^+} \frac{1}{(2-a)^{\frac{1}{a}}}$ ⇒ Let $b = \frac{1}{a}$

as $a \to 0^+$ $b \to \infty$

$\lim_{b\to\infty} \frac{1}{\left(2-\frac{1}{b}\right)^{b}}$ ⇒ $\frac{1}{2^{\text{large}}}$ ⇒ zero.

may be not 2 but certainly bigger than 1 raised to a power makes it get large

4.4

Pg 73 → 77

① $\lim_{x\to\infty} \frac{5x^2+2}{7x^2+3} = \frac{5}{7}$ ② $\lim_{x\to\infty} \frac{6x^2+3}{4x+1} = \lim_{x\to\infty} x \cdot \frac{6x+\frac{3}{x}}{4+\frac{1}{x}} = \infty$

③ $\lim_{x\to-\infty} \frac{6x^3+2x^2+1}{5x^2-1} = -\infty$ ④ $\lim_{x\to\infty} \frac{x^2+\frac{1}{x}}{x^2-\frac{1}{x}} = 1$

⑤ $\lim_{x\to 0} \frac{x^2+\frac{1}{x}}{x^2-\frac{1}{x}} = \lim_{x\to 0} \frac{x^3+1}{x^3-1} = -1$

⑥ $\lim_{x\to 1^+} \frac{x}{x-1} = \lim_{x\to 1^+} \frac{x}{x} \cdot \frac{1}{1-\frac{1}{x}} = +\infty$

⑦ $\lim_{x\to 1^-} \frac{x}{x-1} = -\infty$

⑧ $\lim_{x\to\infty} \frac{x}{x+1} = 1$ $\lim_{x\to-\infty} 6x \Rightarrow \lim_{x}$

⑨ $\lim_{x\to-\infty} 2^x = \lim_{x\to+\infty} \frac{1}{2^x} = 0$ — fastest

Tbl min 1
ΔTbl .01
2nd table, and down and

⑩ $\lim_{x\to\infty} 2^x = \infty$

⑪ $\lim_{x\to 0^+} (2+x)^{\frac{1}{x}} = \infty$ basically $\lim_{N\to\infty} 2^N$??

Tbl min -.5
ΔTbl .01
closer → 0
from the left

⑫ $\lim_{x\to 0^-} (2+x)^{\frac{1}{x}} = 0$ basically $\lim_{N\to\infty} \frac{1}{2^N}$

⑬ $\lim_{x\to\infty} \left[\frac{1}{x-\left(\frac{x^2}{2}\right)} - \frac{1}{x}\right] = \lim_{x\to\infty} \left[\frac{1}{x(1-\frac{x}{2})} - \frac{1}{x}\right]$

$\lim_{x\to\infty} \left[\frac{1-1+\frac{x}{2}}{x(1-\frac{x}{2})}\right] = \lim_{x\to\infty} \frac{x}{2x(1-\frac{x}{2})} = \lim_{x\to\infty} \frac{1}{2(1-\frac{x}{2})}$

$\lim_{x\to\infty} \frac{1}{2-x} = 0$

⑭ $\lim_{x\to 0} \frac{1}{2-x} = \frac{1}{2}$

$2^x - 1$

$\lim_{a \to \infty}$ $\frac{1}{2^a} - 1$

$\frac{1 - 2^a}{2^a}$

$\frac{1}{2^a}$

4.4

15. $\lim_{x\to\infty}\left[\frac{1}{x+1}-\frac{1}{x}\right]=\lim_{x\to\infty}\frac{x-x-1}{x(x+1)}=\lim_{x\to\infty}\frac{-1}{x(x+1)}=0$

16. $\lim_{x\to 0^+}\left[\frac{1}{x+1}-\frac{1}{x}\right]=\lim_{x\to 0^+}\frac{x-x-1}{x(x+1)}=\lim_{x\to 0^+}\frac{-1}{x(x+1)}=-\infty$

17. $\lim_{x\to 0^-}\left[\frac{1}{x+1}-\frac{1}{x}\right]=\lim_{x\to 0^-}\frac{-1}{x(x+1)}=+\infty$

18. $\lim_{x\to\infty}\frac{2^x}{3^x}=0$ 3^x is much larger than 2^x.
$\lim_{x\to\infty}\left(\frac{2}{3}\right)^x$

19. $\lim_{x\to 0^+}\frac{2^x+3}{2^x-1}$ Top $\Rightarrow 5$ bottom $\to 0$
but this cause fraction to become large $\therefore \to\infty$

20. $\lim_{x\to 0^-}\frac{2^x+3}{2^x-1}\to -\infty$ because denominator will be <0.

21. $\lim_{x\to\infty}\frac{2^x+3}{2^x-1}=\lim_{x\to\infty}\frac{1+\frac{3}{2^x}}{1-\frac{1}{2^x}}=1$

22. $\lim_{x\to-\infty}\frac{2^x+3}{2^x+1}=\lim_{a\to+\infty}\frac{\frac{1}{2^a}+3}{\frac{1}{2^a}+1}=\frac{3}{1}=3$,

23. $\lim_{x\to\infty}\left[\frac{1}{x-1}-\frac{1}{x^2-1}\right]=\lim_{x\to\infty}\left[\frac{x^2-1-x+1}{(x-1)(x^2+1)}\right]=$

$\lim_{x\to\infty}\frac{x^2-x}{(x-1)(x^2-1)}=\lim_{x\to\infty}\frac{x}{x^2-1}=\lim_{x\to\infty}\frac{\frac{1}{x}}{1-\frac{1}{x^2}}=0$

24. $\lim_{x\to 1^+}\left[\frac{1}{x-1}-\frac{1}{x^2-1}\right]=\lim_{x\to 1^+}\frac{x}{x^2-1}=\frac{1}{0}=$ diverges to ∞

77

25 $\lim_{x\to 1^+} \frac{\sqrt{x^2-1}}{\sqrt{x^3-1}} = \lim_{x\to 1} \sqrt{\frac{(x-1)(x+1)}{(x-1)(x^2+x+1)}} = \lim_{x\to 1^+} \sqrt{\frac{x+1}{x^2+x+1}}$

$\sqrt{\frac{2}{3}} = \frac{\sqrt{6}}{3}$

26. $\lim_{x\to\infty} \frac{x}{\sqrt{2x^2+1}} = \lim_{x\to\infty} \sqrt{\frac{x^2}{2x^2+1}} = \lim_{x\to\infty} \sqrt{\frac{1}{2+\frac{1}{x^2}}} = \sqrt{\frac{1}{2}}$

$\frac{\sqrt{2}}{2}$

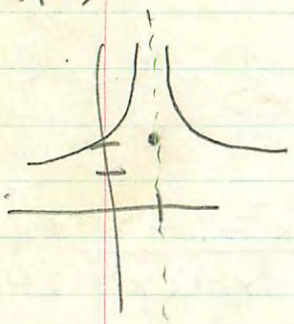
27.

28. $\lim_{x\to 0} (1+3x)^{\frac{2}{x}} = \lim_{x\to 0} \left((1+3x)^{\frac{1}{3x}}\right)^{2\cdot 3} = e^6$

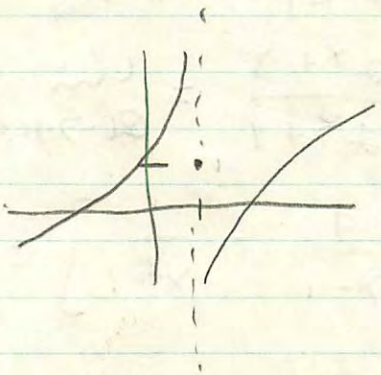
29. $\lim_{x\to 0} (1+x^2)^{\frac{1}{x}} = \lim_{x\to 0} \left((1+x^2)^{\frac{1}{x^2}}\right)^{x} = e^x = 1$

30. $\lim_{n\to 0} \left(1+\frac{4}{x}\right)^{\frac{x}{4}} = e$ for fixed x.

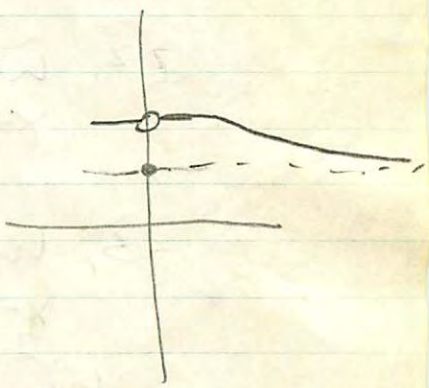
31.



32.



33.



34. a) $f(x)=3x \quad g(x)=x$

b) $f(x)=x^2 \quad g(x)=x$

c) $f(x)=x \quad g(x)=x^2$

#35 pg 77

$0 < x < 1$

$0 < 1 < \frac{1}{x}$

a) $\frac{1}{x}$ is a #. There is an integer bigger than $\frac{1}{x}$. There is a consecutive integer smaller than $N+1$ which will be less than or equal to $\frac{1}{x}$.

$N+1 > \frac{1}{x} \geq N$

b) WTS $(1+x)^{\frac{1}{x}} \leq \left(1+\frac{1}{N}\right)^{\frac{1}{x}} < \left(1+\frac{1}{N}\right)^{N+1}$

$N+1 > \frac{1}{x}$ $\quad \frac{1}{x} \geq N$

$x > \frac{1}{N+1}$ $\quad \frac{1}{N} \geq x$

$\frac{1}{N+1} < x < \frac{1}{N}$

add 1

$1 + \frac{1}{N+1} < x+1 \leq \frac{1}{N} + 1$

$(x+1)^{\frac{1}{x}} \leq \left(\frac{1}{N}+1\right)^{\frac{1}{x}}$ raised to = power

$(x+1)^{\frac{1}{x}} \leq \left(\frac{1}{N}+1\right)^{\frac{1}{x}} < \left(\frac{1}{N}+1\right)^{N+1}$ raised to bigger power.

35 $0<x<1 \Rightarrow 1<\frac{1}{x}$

? If $\frac{1}{x}$ is not an integer, there are 2 consecutive integers, N & $N+1$ where $N \le \frac{1}{x} < N+1$

a) WTS $N+1 > \frac{1}{x} > N$

$\frac{1}{N+1} < x \le \frac{1}{N}$

b) $(1+x) \le (1+\frac{1}{N})$ so $(1+x)^{\frac{1}{x}} \le (1+\frac{1}{N})^{\frac{1}{x}}$

Since $\frac{1}{x} < N+1$

$(1+x)^{\frac{1}{x}} < (1+\frac{1}{N})^{\frac{1}{x}} \le (1+\frac{1}{N})^{N+1}$

$(1+x)^{\frac{1}{x}} < (1+\frac{1}{N})^{1/x} \le (1+\frac{1}{N})(1+\frac{1}{N})^{N}$

c) WTS $(1+x)^{\frac{1}{x}} > \dfrac{(1+\frac{1}{N+1})^{N+1}}{1+\frac{1}{N+1}}$ Sets up similarity b above.

$\frac{1}{N+1} < x$

$1+\frac{1}{N+1} < 1+x$

$(1+\frac{1}{N+1})^{\frac{1}{x}} < (1+x)^{\frac{1}{x}}$ Since $\frac{1}{x} > N$

$(1+\frac{1}{N+1})^{N} < (1+\frac{1}{N+1})^{\frac{1}{x}} < (1+x)^{\frac{1}{x}}$

$\dfrac{(1+\frac{1}{N+1})^{N}(1+\frac{1}{N+1})}{1+\frac{1}{N+1}} < (1+\frac{1}{N+1})^{\frac{1}{x}} < (1+x)^{\frac{1}{x}}$

$\dfrac{(1+\frac{1}{N+1})^{N+1}}{1+\frac{1}{N+1}} < (1+x)^{\frac{1}{x}}$

Cont ↓

$$\frac{\left(1+\frac{1}{N+1}\right)^{N+1}}{1+\frac{1}{N+1}} < (1+x)^{\frac{1}{x}} \leq \left(1+\frac{1}{N+1}\right)^{N+1}$$

Now take $\lim_{x\to 0}$. Notice we have $\lim_{a\to\infty}\left(1+\frac{1}{a}\right)^a = e$

as $x \to 0$ $a \to +\infty$

Since ~~$x \to N$~~ $\frac{1}{x} \geq N$ $N \to \infty$

$$\frac{e}{1} < \lim_{x\to 0}(1+x)^{\frac{1}{x}} < e$$

$\lim_{N\to\infty}\left(1+\frac{1}{N+1}\right)^{N+1}$

Indeed we are "sandwiching" $\lim_{x\to 0}(1+x)^{\frac{1}{x}}$ between e and e $\therefore$ it must be e.

c) $(1+x)^{\frac{1}{x}} > \dfrac{\left(1+\frac{1}{N+1}\right)^{N+1}}{1+\frac{1}{N+1}}$

from * $\quad \frac{1}{N+1} < x$

$1+\frac{1}{N+1} < x+1$

$N < \frac{1}{x}$ smaller power

$\left(1+\frac{1}{N+1}\right)^{N} < \left(1+\frac{1}{N+1}\right)^{\frac{1}{x}} < (x+1)^{\frac{1}{x}}$

$\dfrac{\left(1+\frac{1}{N+1}\right)^{N}\left(1+\frac{1}{N+1}\right)}{\left(1+\frac{1}{N+1}\right)} <$ multiply by 1 $<$

$\dfrac{\left(1+\frac{1}{N+1}\right)^{N+1}}{1+\frac{1}{N+1}} < (x+1)^{\frac{1}{x}}$

d. Put b & c together

$\dfrac{\left(1+\frac{1}{N+1}\right)^{N+1}}{1+\frac{1}{N+1}} < (x+1)^{\frac{1}{x}} < \left(\frac{1}{N}+1\right)^{N+1}$ Now take limit.

$\left(\frac{1}{N}+1\right)^{N}\left(\frac{1}{N}+1\right)$

$\dfrac{e}{1+0} < \lim_{x\to\infty} (x+1)^{\frac{1}{x}} < e\,(0+1)$

conclude $\uparrow$ is sandwiched between e and e.